Interaction graph estimation in a neural network

Brochini, Galves, Hodara, Ost, Pouzat

Introduction

A model for neural networks activity introduced by Galves and Löcherbach (2013)

An interaction graph estimation procedure proposed by Duarte, Galves, Löcherbach and Ost (2016).

Application on real data and simulations with additional theoretical results. This requires a pre-treatment of the raw data called spike sorting.

The model

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For all finite subset $J \subset I$,

$$P\left(X_t(j) = x_j,\, j \in J / X_{-\infty}^{t-1}(I)\right) = \prod_{j\in J} P\left(X_t(j) = x_j / X_{-\infty}^{t-1}(I)\right).$$

The model

Let L_t^i be the last spike time of neuron i before time t defined by :

$$L_t^i = \sup\{s < t : X_s(i) = 1\}.$$

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Each neuron $i \in I$ possesses an interaction neighborhood $V_i \subset I$ satisfying for all $A \subset I$ with $V_i \subset A$,

$$P\left(X_t(i) = x_i / X_{L_t^i}^{t-1}(A)\right) = P\left(X_t(i) = x_i / X_{L_t^i}^{t-1}(V_i)\right).$$

The sample

neurons

l	0	0	0	1	0	0	1	0	1	0	0	0	0	0	0	1
k	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	1
j	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0
i	0	1	0	0	0	0	1	0	0	0	0	1	0	0	0	1

time

FIGURE: Sample of X process with size n

Decision rule

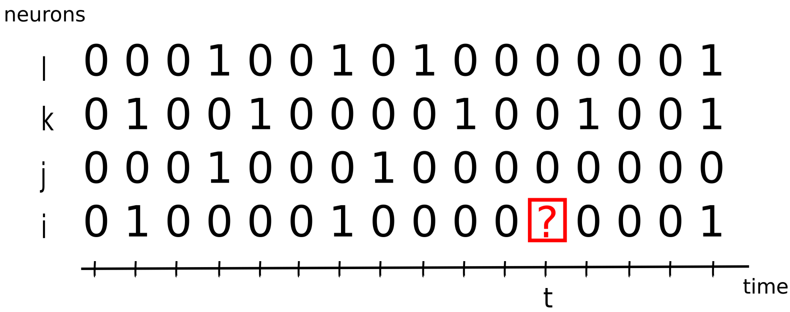

FIGURE: What is the probability of neuron i to spike at time t ?

Decision rule

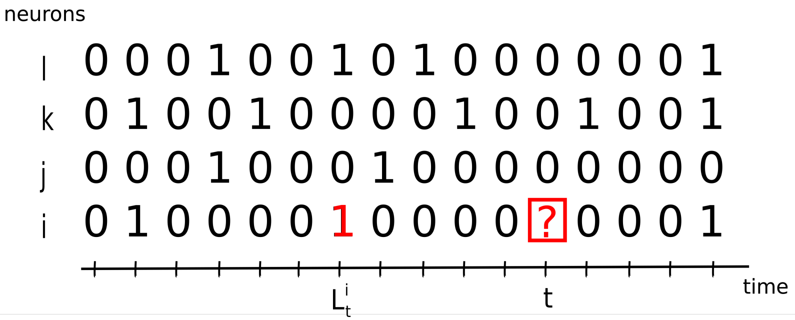

FIGURE: Last time neuron i spiked before t :
$L_t^i = sup\{s < t : X_s(i) = 1\}$

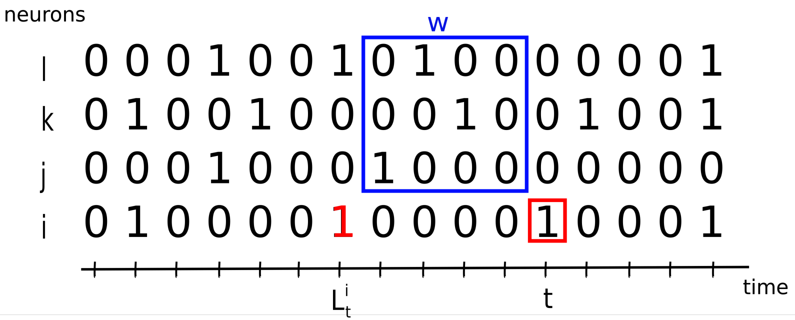

FIGURE: The pattern w is the portion of the past containing the information for the decision of the state of neuron i at time t.

Does k influences i ?

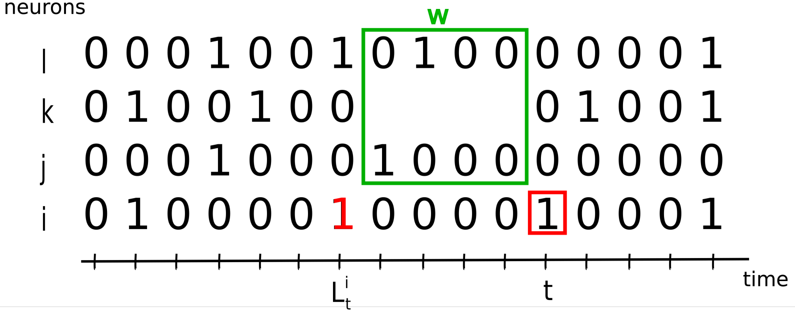

FIGURE: If k influences i, then a modification of the pattern concerning only k should lead to a different realisation for i in the following time step.

Empirical probability

For a given neuron i and a given pattern w, we denote by $N_i(w)$ the number of occurrences of w, and by $N_i(w, 1)$ the number of occurrences of w followed by a spike of neuron i.
For w such that $N_i(w) > n^{\xi + \frac{1}{2}}$ for some parameter $\xi \in]0, \frac{1}{2}[$, we compute

$$\hat{p}_{(i,n)}(1/w) = \frac{N_i(w, 1)}{N_i(w)}.$$

Does k influences i ?

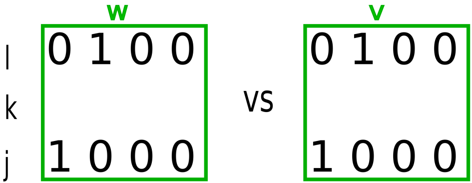

FIGURE: We will compare the empirical probabilities for couples of patterns that are identical outside k.

The sensitivity parameter ϵ.

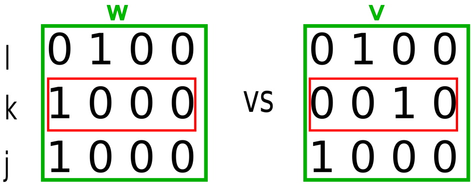

FIGURE: If $|\hat{p}_{(i,n)}(1|w) - \hat{p}_{(i,n)}(1|v)| > \epsilon$ for any couple (w, v) satisfying $w \setminus \{k\} = v \setminus \{k\}$ we accept k in the estimated interaction neighborhood.

Results and discussion

- simulation to explore sensitivity and cutoff parameters
- procedure to deal with small sample sizes
- issue of partially observed networks
- experimental results
- limitations of the method

Exploring parameters ϵ and ξ with simulations

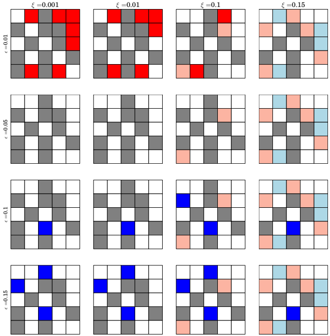

FIGURE: Color code : *white*− correct absent, *grey*− correct present, *red*− false positive, *blue*− false negative. *light* − *blue* and *light* − *red* inconclusive (Network 1 with $\mu = 1$ and $n = 1e5$). Original Duarte et al estimator

Pruning procedure

- Limitation : Small sample size and/or greater number of neurons : Too many inconclusives

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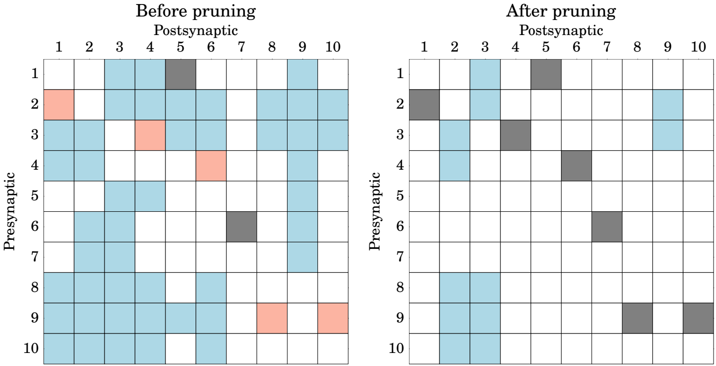
- Limitation : Small sample size and/or greater number of neurons : Too many inconclusives
- Pruning procedure : re-estimate graph disconsidering neurons that the original estimator says is NOT presynaptic
- pruning procedure is due to the reduction in the number of presynaptic candidate neurons while maintaining the same sample size, leading to the improvement of the estimation quality.
- Analyical results : iterative pruning procedure conserves the consistency of the estimation

Pruning procedure



Estimation on a partially observed network

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- A key assumption in Duarte et al when determining the neighbourhood of neuron i is that we have access to the activity of *all* neurons presynaptic to i
- Not realistic experimentally !
- What happens when this condition is not met ?
- *Numerical experimentation* : estimated the connection graph of subgraphs compared to true connections yielded :
  - All connections identified as false were indeed false
  - All connections identified as true were either indeed true or a false positive due to a projected connection
- Analytical Results : guarantee that if there is NO PATH between neuron j and i involving an unobserved neuron, then the estimator is not expected to produce a connection.

Estimation on a partially observed network

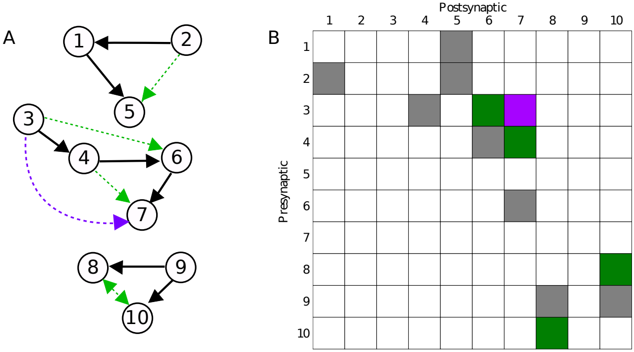

FIGURE: Network 2. Complete graph recovered by procedure that identifies false positives due to projections. This procedure can be used to deal with large N.

- Binning : Spike train $\rightarrow$ symbolic sequence of 0's and 1's : divide sample in small and attribute 1 when a spike occurs inside that window
- Cannot be too small : not enough repetitions of patterns
- Cannot be too large : two spikes of the same neuron in the same window
- Choose the smallest possible window allowing at most 1% superpositions

Results on real data

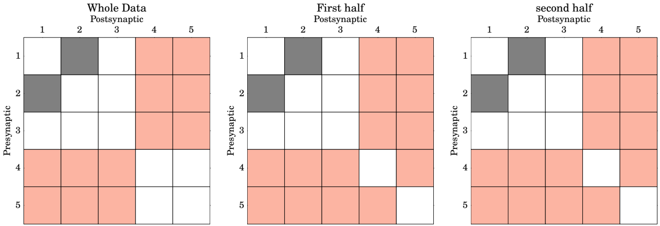

FIGURE: Estimated connection graph for spontaneous activity of projection neurons of the anennal lobe of *Schistocerca americana*.

Method Limitations

- Results concerning partialy observed networks require stationary
- requires huge amount of pattern repetitions : Strong connections and sparse activity is a bad combo !

Estimation of neuronal interaction graph from spike train data
Ludmila Brochini, Antonio Galves, Pierre Hodara, Guilherme Ost, Christophe Pouzat
https://arxiv.org/abs/1612.05226
Thank You !