A test of hypothesis for random graphs with an application in neuroscience

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1st Neuromat Workshop

Main goal

Given two samples of random graphs we want to test if they come from the same probability distribution.

$$\begin{cases} H_0 : \pi = \pi' \\ H_1 : \pi \neq \pi' \end{cases}$$

Inspiration

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TESTING STATISTICAL HYPOTHESIS ON RANDOM TREES AND APPLICATIONS TO THE PROTEIN CLASSIFICATION PROBLEM[1]

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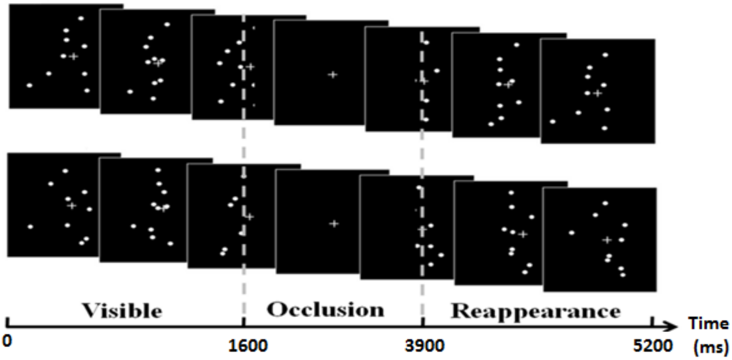
Test statistic

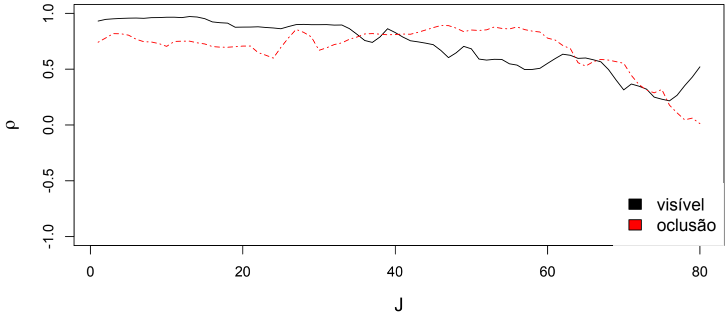
Given two samples of random graphs $\mathbf{G_1} = (g_{11}, \ldots, g_{1n})$ and $\mathbf{G_2} = (g_{21}, \ldots, g_{2m})$ of size n and m, respectively, we define the statistic

$$W(\mathbf{G_1}, \mathbf{G_2}) = \sup_{g \in \mathbb{G}(V)} |\overline{d}_{\mathbf{G_1}}(g)) - \overline{d}_{\mathbf{G_2}}(g))|$$

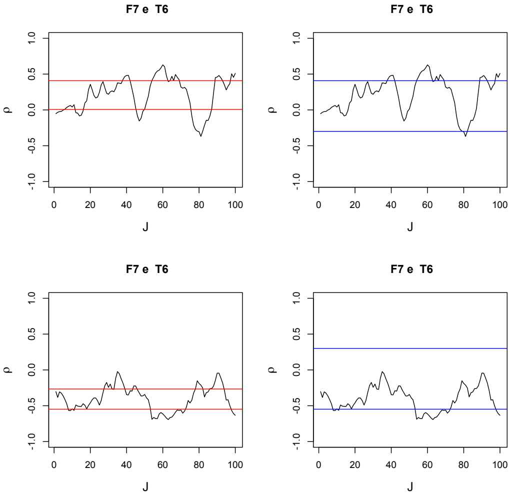
Application on random graphs from EEG data



Application on random graphs from EEG data



Application on random graphs from EEG data



Application on random graphs from EEG data

$q_1^{r,ij}$, $q_3^{r,ij}$: first and third quartiles of the correlations

$(\rho_{ij}^{r,J_1}, \rho_{ij}^{r,J_2}, \ldots, \rho_{ij}^{r,J_M})$.

$0 \leq c \leq 1$.

$$X_{ij}^{r,J} = \begin{cases} 1, & \text{if } \rho_{ij}^{r,J} \geq \max(c, q_3^{r,ij}) \text{ or } \rho_{ij}^{r,J} \leq \min(-c, q_1^{r,ij}) \\ 0, & c.c. \end{cases} \tag{1}$$

Application on random graphs from EEG data

Biological vs non-biological		Window				
	c	1	21	41	61	81
Visible	0.3	0.1842	0.4639	0.0723	0.4606	-
	0.4	0.0899	0.6914	0.0373	0.7074	-
	0.5	0.1014	0.6621	0.0295	0.5910	-
	0.6	0.0781	0.5470	0.0437	0.6122	-
	0.7	0.0915	0.5578	0.0836	0.5448	-
	0.8	0.0373	0.6539	0.1819	0.4467	-
Occlusion	0.3	0.7434	0.8548	0.2557	0.1682	0.0251
	0.4	0.6241	0.9157	0.4173	0.1200	0.0117
	0.5	0.8227	0.8816	0.3764	0.1292	0.0185
	0.6	0.9421	0.9028	0.3456	0.0778	0.0433
	0.7	0.9706	0.8289	0.4190	0.0441	0.0542
	0.8	0.7486	0.2685	0.4036	0.1252	0.0851

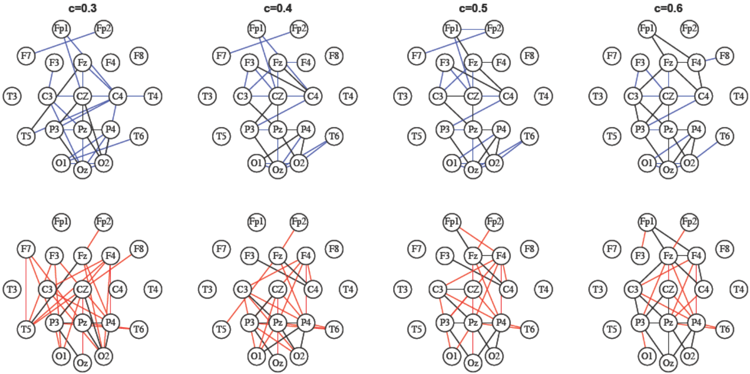
P-value for visible (biological vs non-biological) and occlusion (biological vs non-biological) fase.

Application on random graphs from EEG data

Visible vs occlusion		Window			
	c	1	21	41	61
Biological	0.3	0.0059	0.4444	0.2322	0.0106
	0.4	0.0030	0.5836	0.2675	0.0204
	0.5	0.0019	0.4294	0.1984	0.0278
	0.6	0.0035	0.3827	0.3037	0.0563
	0.7	0.0019	0.4352	0.4513	0.0548
	0.8	0.0013	0.534	0.5022	0.1338
Non-biological	0.3	0.0024	0.7959	0.0407	0.6918
	0.4	0.0016	0.8777	0.0802	0.5380
	0.5	0.0016	0.8278	0.1249	0.6673
	0.6	0.0014	0.6362	0.2222	0.5629
	0.7	0.0023	0.6046	0.1686	0.3582
	0.8	0.0066	0.5784	0.2959	0.3360

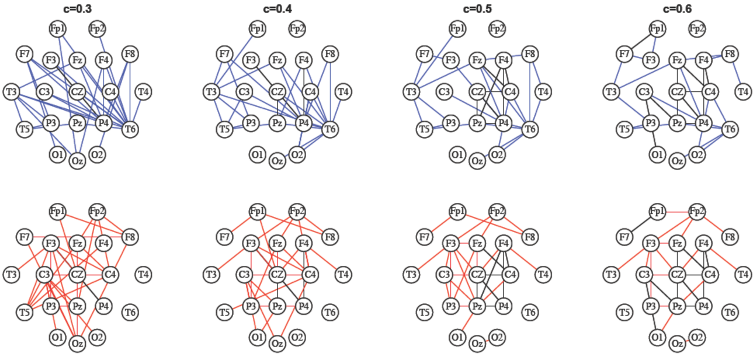
P-value for biological movement (visible vs oclusion) and non-biological movement (visible vs occlusion) experiment.

Application on random graphs from EEG data



30 more frequent edges for visible (blue) and occlusion (red) for biological movement in time 1 (black = common edges)

Application on random graphs from EEG data



30 more frequent edges for visible (blue) and occlusion (red) for biological movement in time 61 (black = common edges)