Self-Organized Criticality and Neuronal Avalanches in Networks with Dynamical Synapses

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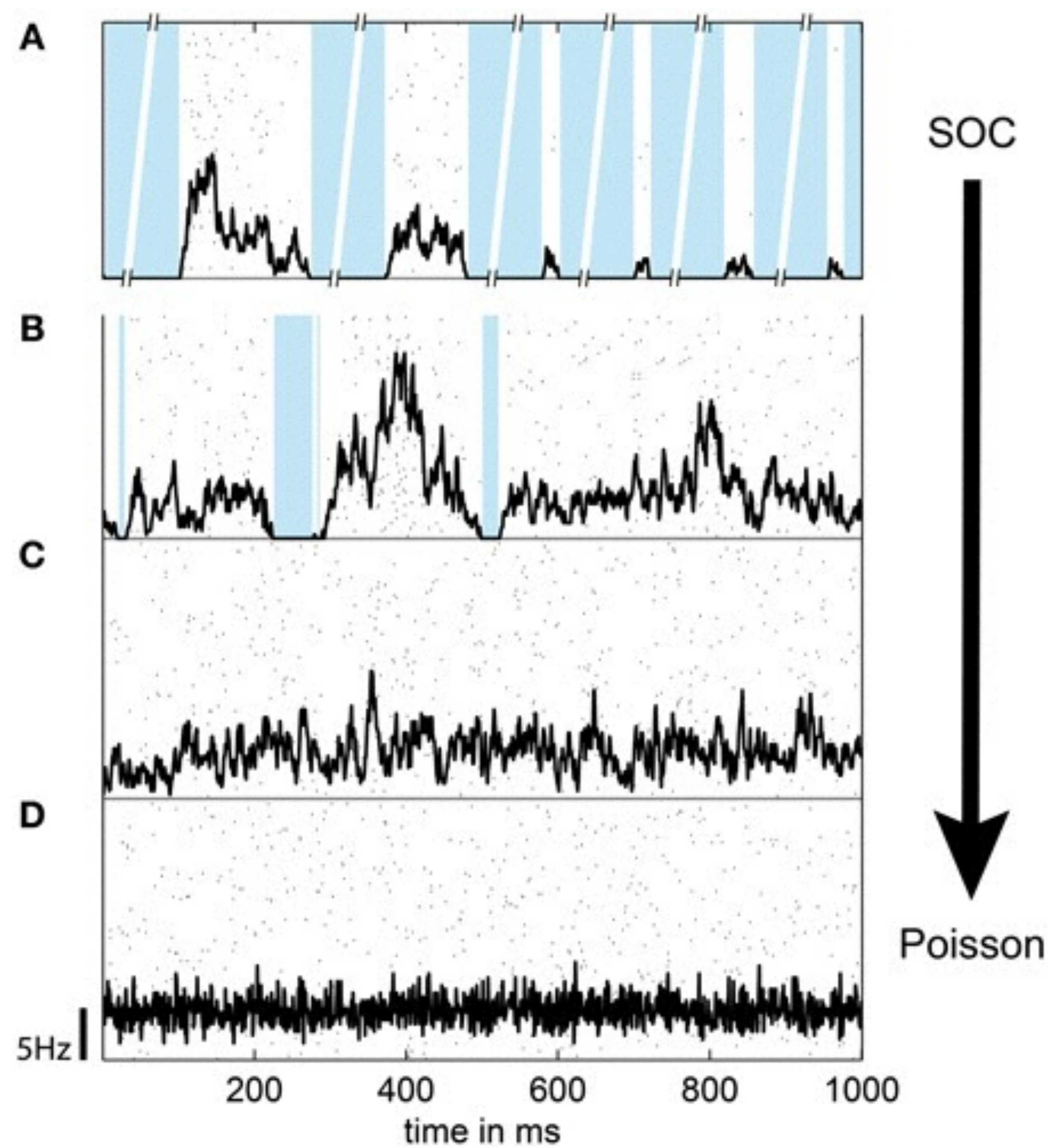
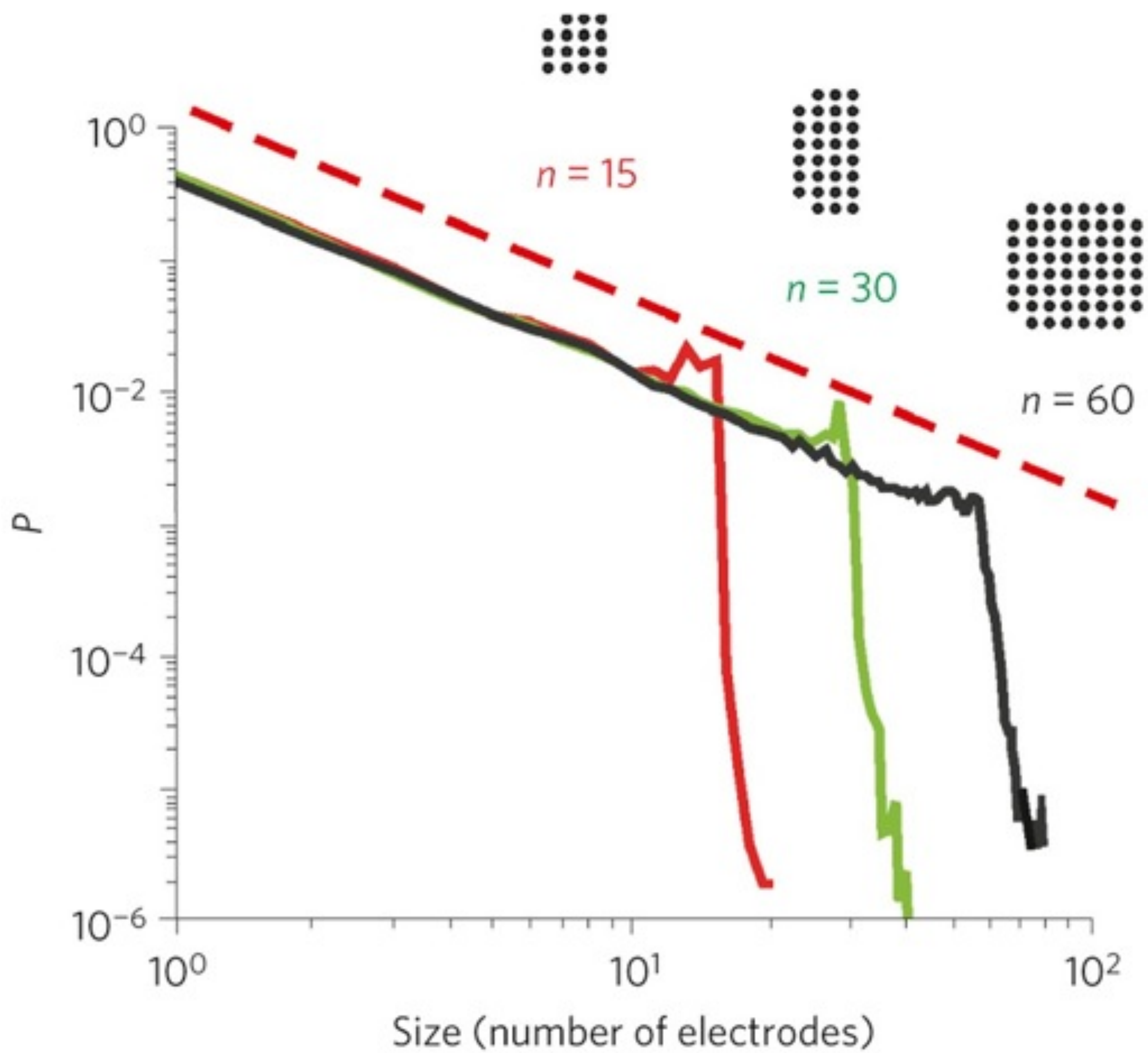
Motivation: neuronal avalanches



Size distribution of neuronal avalanches



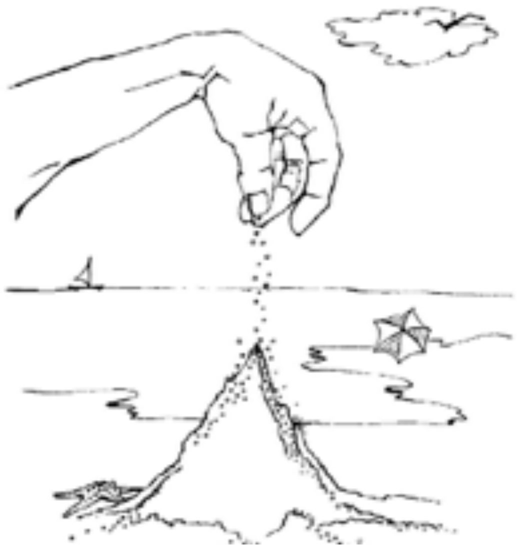
$$P(s) = c\ s^{-3/2}$$

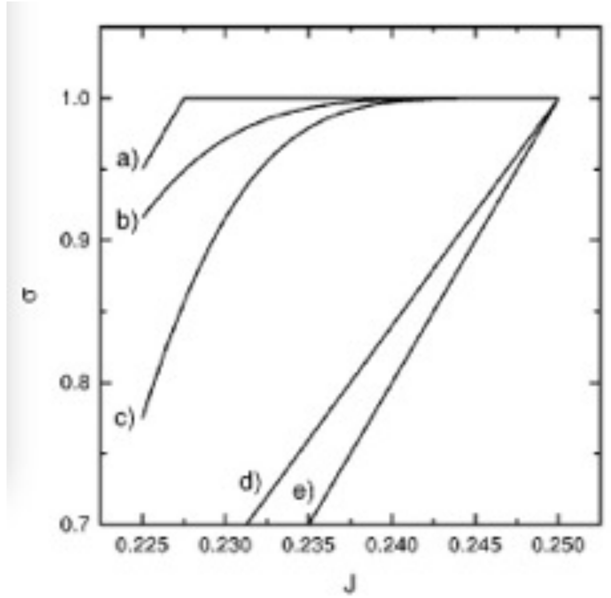
Quick historical background of SOC

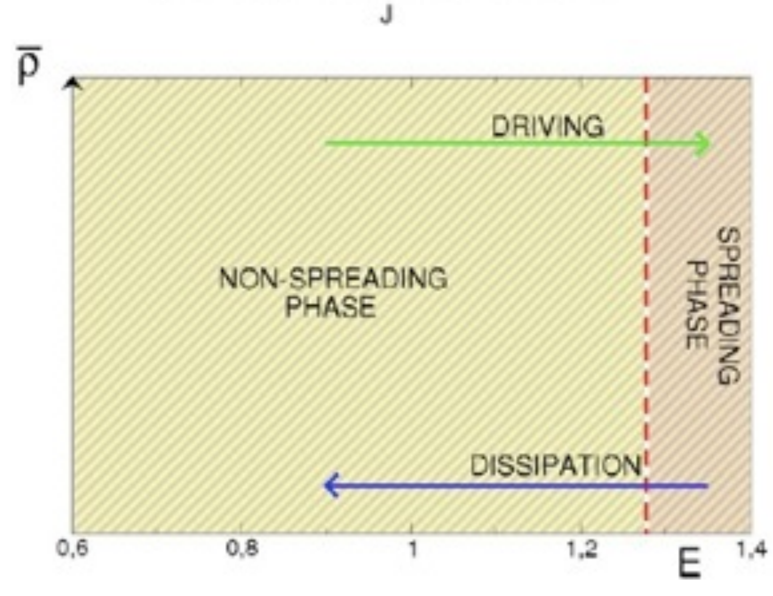
- Microscopically conservative models with true SOC
  (Bak, Tang & Wiesenfeld *PRL* 87, Jensen *SOC* 98)

- Non-conservative models with after-avalanche (hard) loading, presenting pseudo-SOC
  (Drossel *PRL* 92, Kinouchi & Prado, *PRE* 99)

- Non-conservative models with intra-avalanche (soft) loading, presenting SOqC (self-organized quase-criticality)
  (Levina, Herrmann & Geisel *Nat Phys* 07, Bonachela et al *JSTAT* 09, 10)







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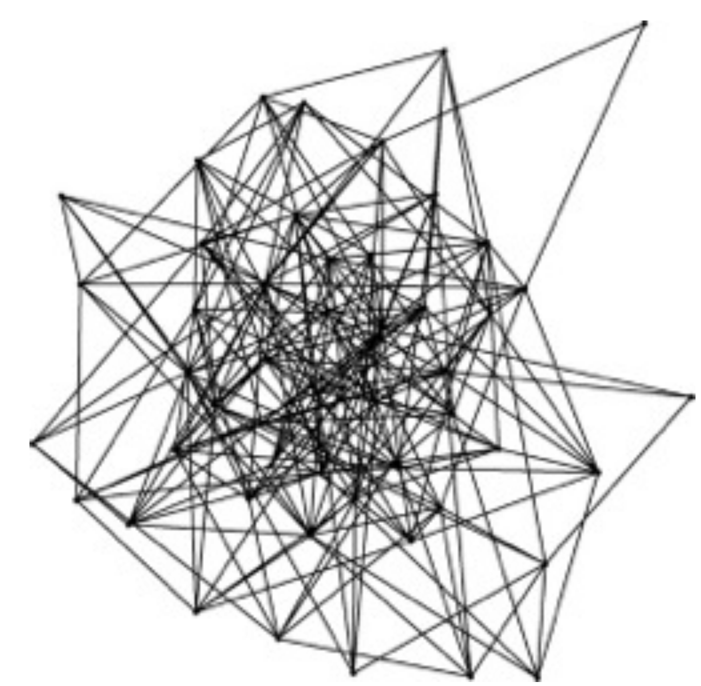
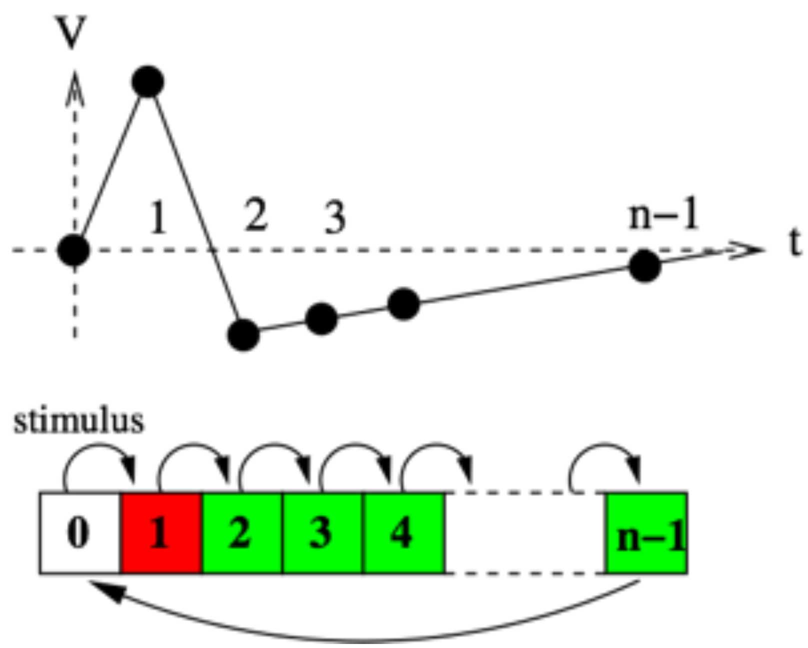
Here:

- Microscopically non-conservative model with "soft" loading, but
- Probabilistic cellular automata (larger system sizes & easier mean field)
- Finite connectivity (random graph)
- Stationary regime: conservative on average
- Fluctuations vanish when $N \to \infty$



Random network of excitable cellular automata



Control parameter:

Branching ratio:

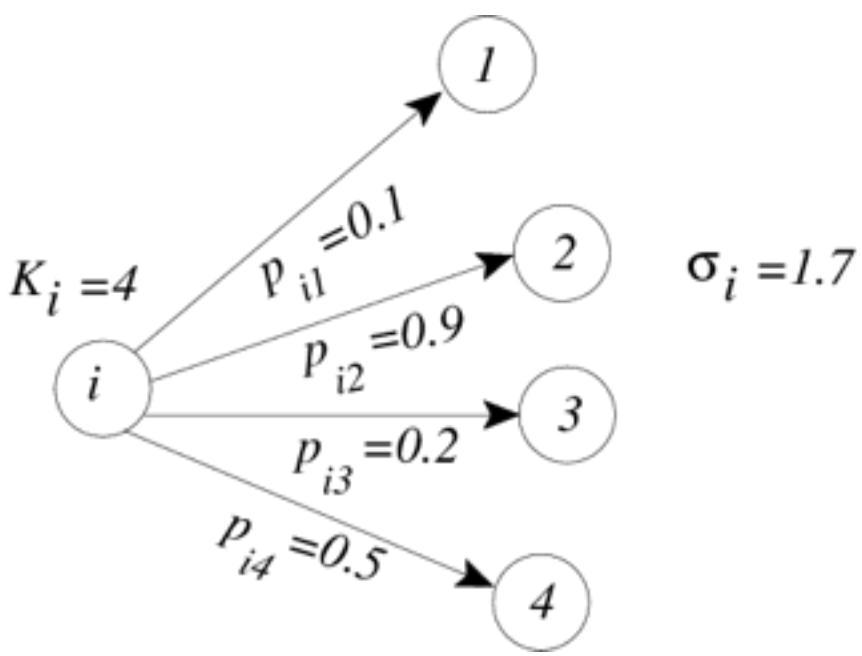
$$\sigma = \langle \sigma_i \rangle = \left\langle \sum_j^K P_{ij} \right\rangle$$

Order parameter:

Mean firing rate:

$$\rho = \lim_{t\to\infty} \rho(t) = \lim_{t\to\infty} \frac{1}{N} \sum_{i=1}^{N} \delta(s_i(t), 1)$$

Random graph with K outcoming synapses

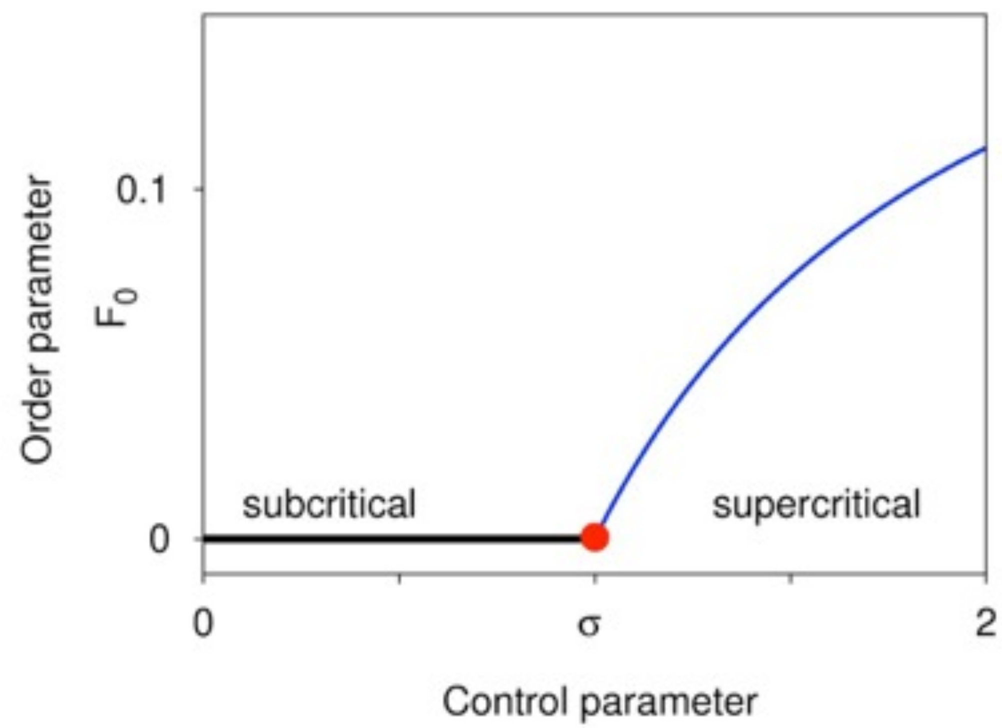


Mean-field equation: (I)

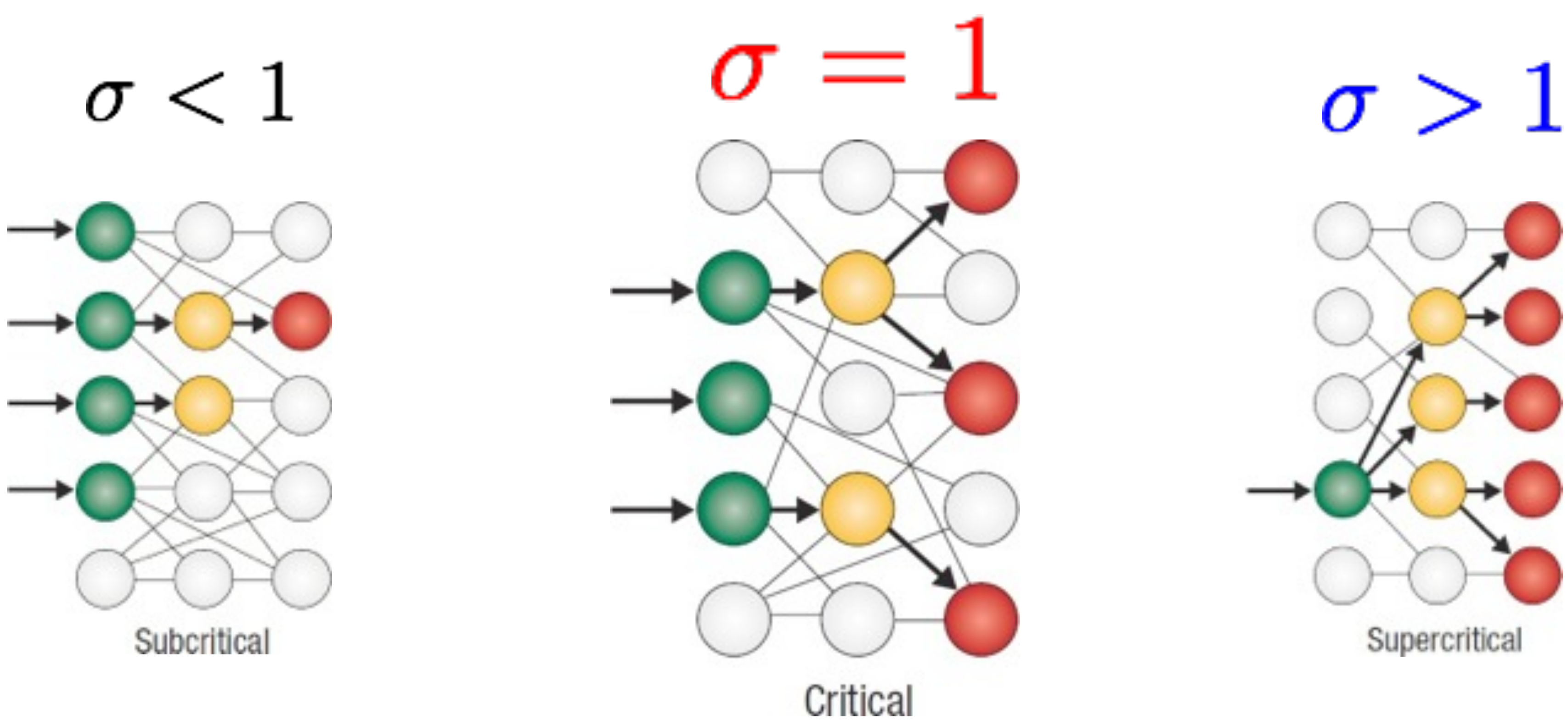
$$\rho = \overbrace{(1-(n-1)\rho)}^{\mathrm{P(quiet)}} \times \left[1 - \underbrace{(1-\sigma\rho/K)^K}_{\mathrm{P(!\ from\ neighbors)}} \times \underbrace{(1-p_h)}_{\mathrm{P(!\ external)}}\right]$$

Kinouchi & Copelli, *Nat. Phys.* 06

Directed-percolation phase transition at $\sigma = \sigma_c = 1$



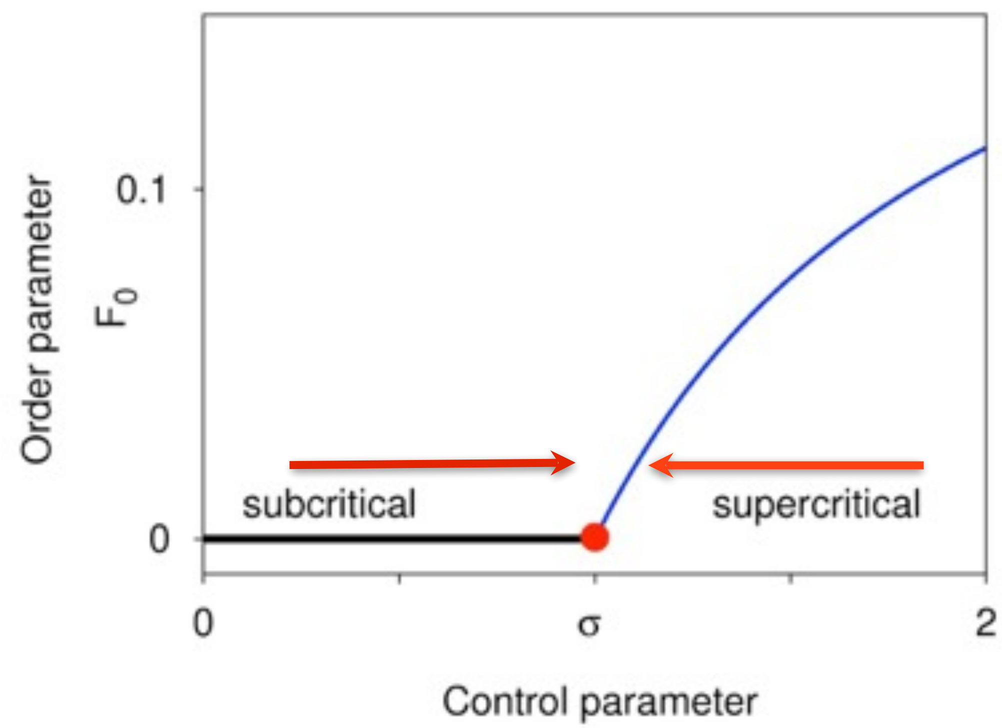
Kinouchi & Copelli, *Nat. Phys.* 06



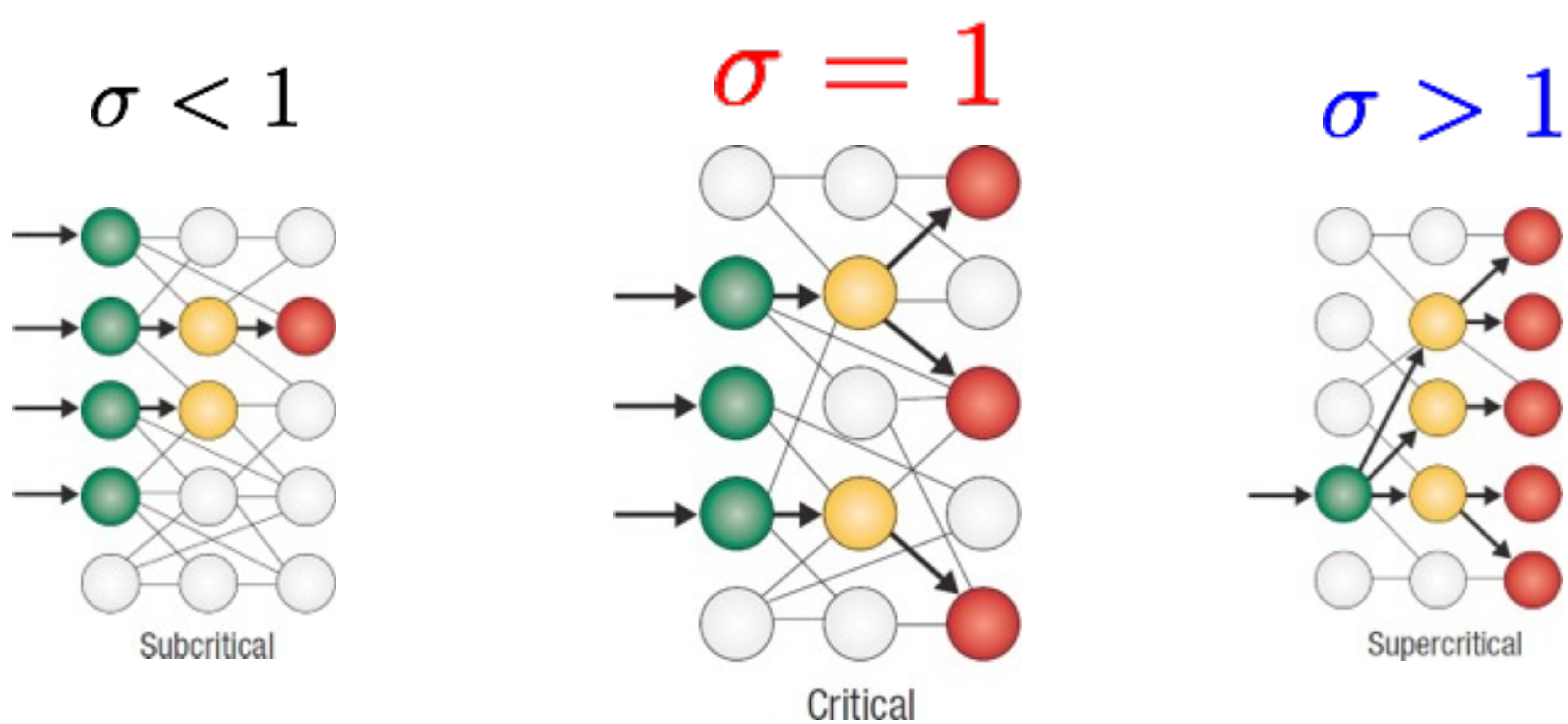
Chialvo, *Nat. Phys.* 06

Directed-percolation phase transition at $\sigma = \sigma_c = 1$

Can synaptic depression tune $\sigma \to \sigma_c = 1$? If so, is it SOC?



Kinouchi & Copelli, *Nat. Phys.* 2006



Chialvo, *Nat. Phys.* 2006

Synaptic Depression

Levina et al, *Nat. Phys.* 07 (LHG)

- Integrate and fire neurons (deterministic)
- all-to-all coupling

$$\dot{h}_i = I_{ext}\delta_{i,\xi_\tau} + \frac{1}{N}\sum_{j=1}^{N} uJ_{ij}\delta\left(t - t_j - \tau_d\right)$$

Synaptic depression: slow load

$$\dot{J}_{ij} = \frac{1}{\tau\nu N}\left(\frac{\alpha}{u} - J_{ij}\right) - uJ_{ij}\delta\left(t - t_j\right)$$

Here

- Cellular automata (probabilistic)
- random graph

$$P\left[s_j(t+1) = 1\right] = P\left[s_j(t) = 0\right] \times \left\{1 - \prod_j^K \left[1 - P_{ij}\delta(S_i(t), 1)\right]\right\}$$

Synaptic depression: slow load

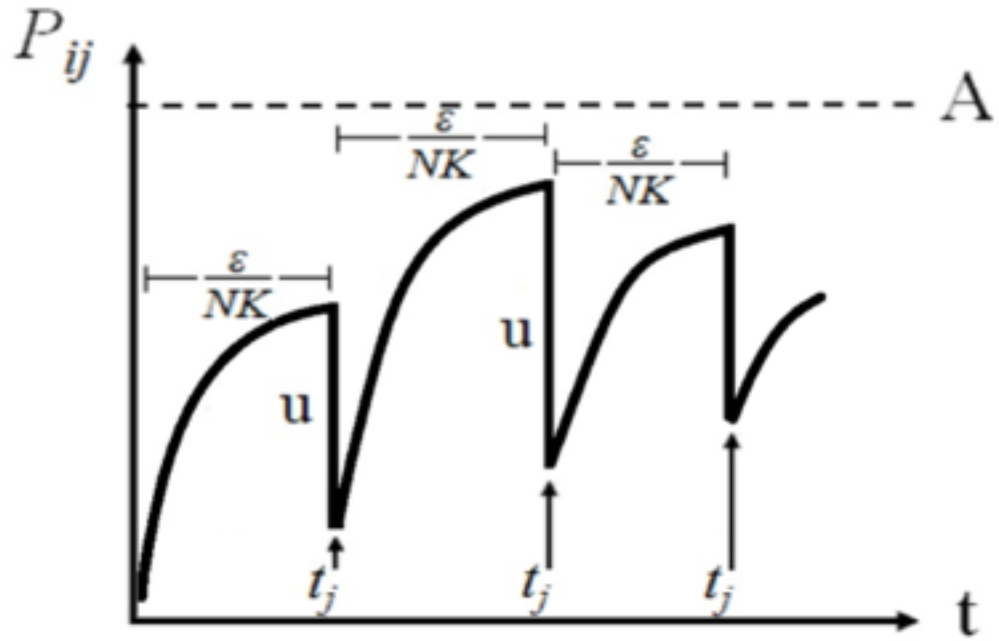
$$P_{ij}(t+1) = P_{ij}(t) + \frac{\epsilon}{NK}(A - P_{ij}) - uP_{ij}(t)\delta(t, t_i)$$

$$0 < A \leq 1$$

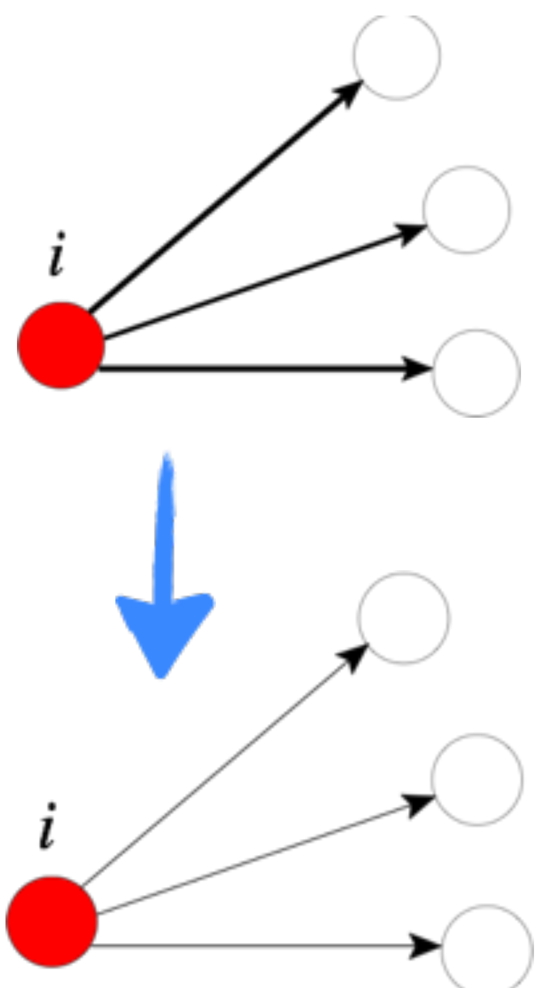
$$0 < u \leq 1$$

Synaptic Depression: two variants

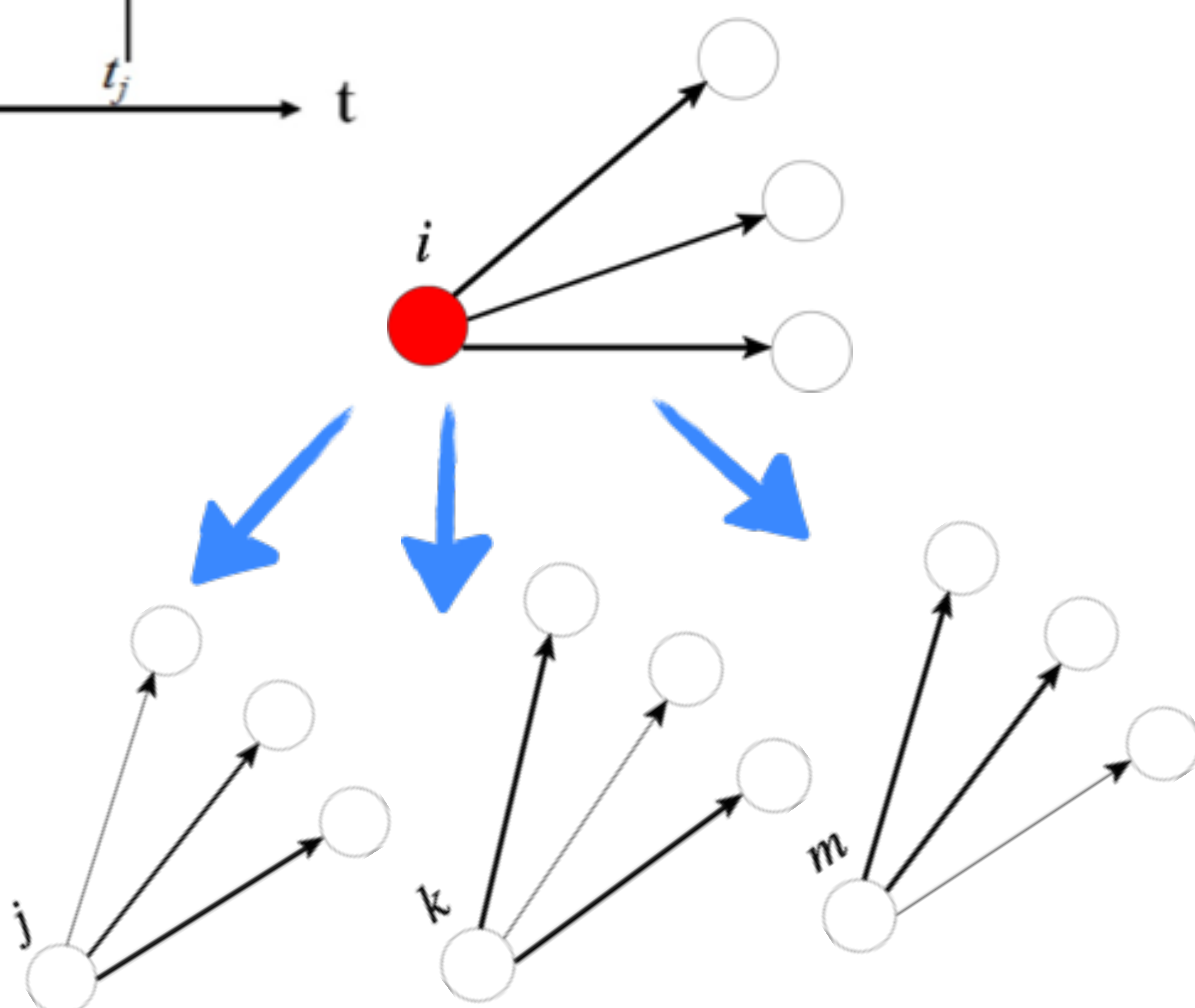
$$P_{ij}(t+1) = P_{ij}(t) + \frac{\epsilon}{NK}(A - P_{ij}) - uP_{ij}(t)\delta(t, t_j)$$



Quenched



Annealed (random neighbor)



Synaptic Depression: two variants

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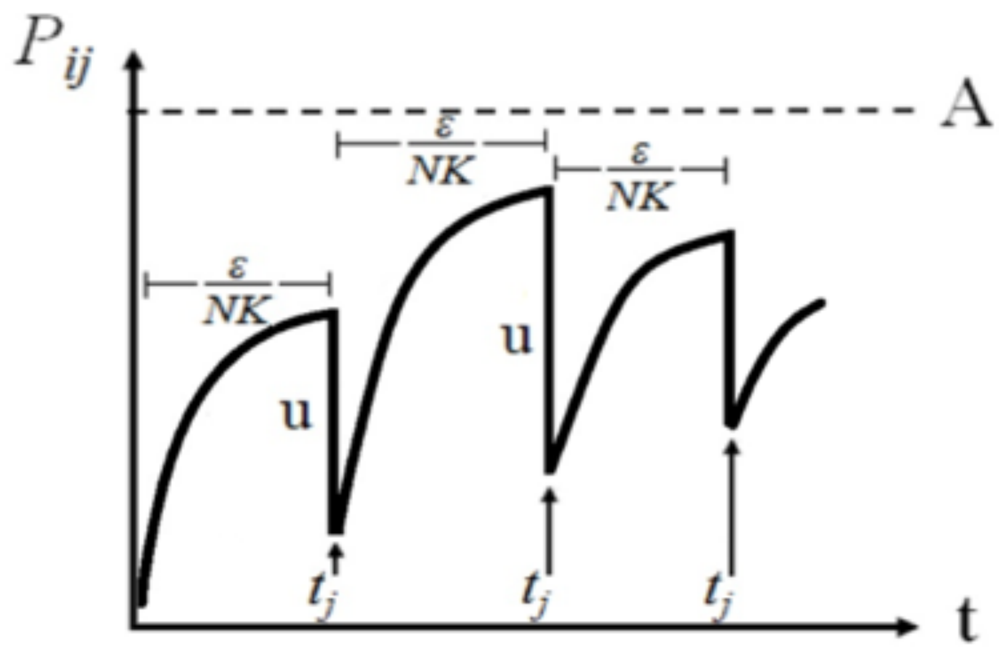

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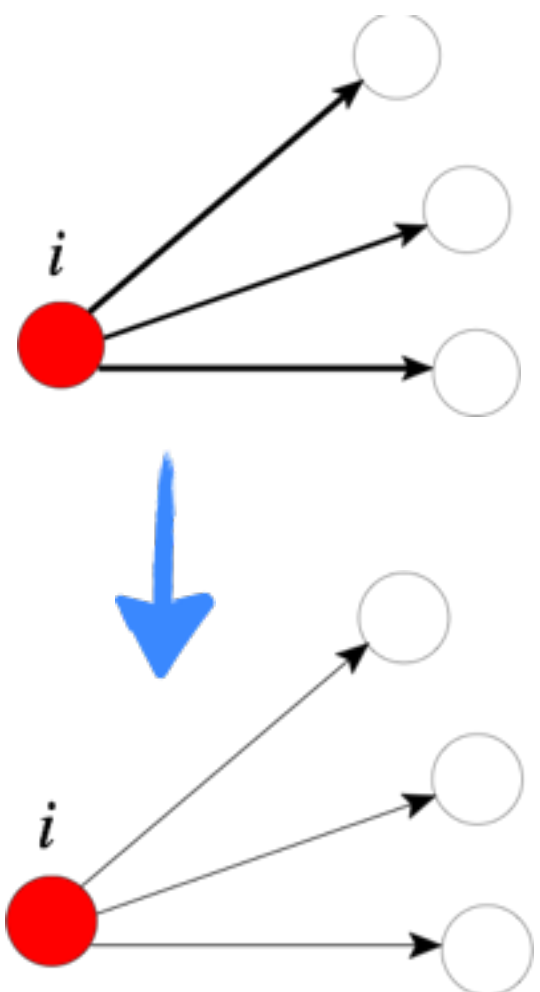

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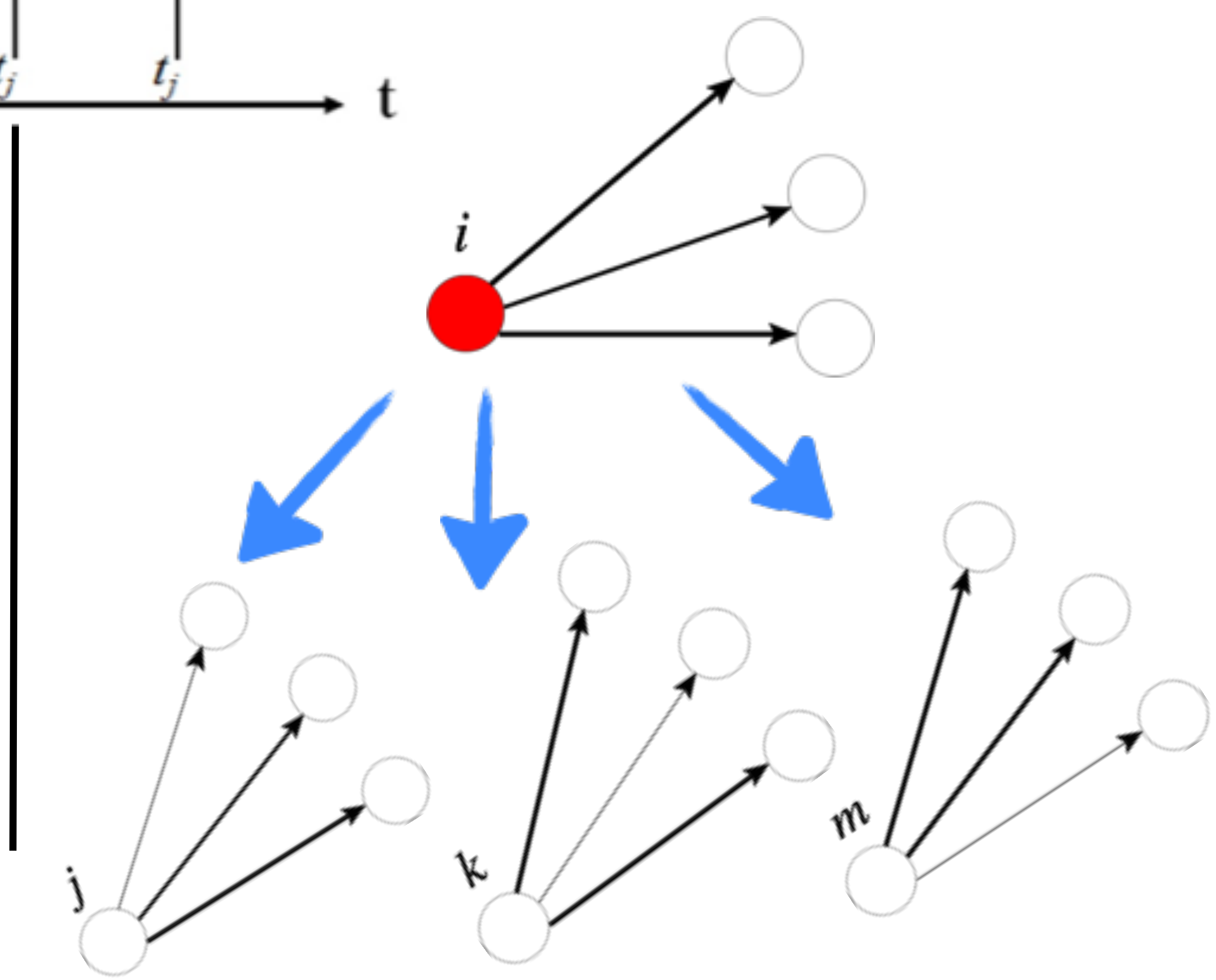


Results

- **Analytical (mean-field theory)**
- Simulations

Mean-field theory

$$P\left[s_j(t+1)=1\right] = P\left[s_j(t)=0\right] \times \left\{1 - \prod_{j}^{K}\left[1 - P_{ij}\delta(S_i(t),1)\right]\right\}$$

Mean-field equation (I)

$$\rho^* = \overbrace{(1-(n-1)\rho^*)}^{\mathrm{P(quiet)}} \times \left[1 - \underbrace{(1-\sigma^*\rho^*/K)^K}_{\mathrm{P(!\ from\ neighbors)}} \times \underbrace{(1-p_h)}_{\mathrm{P(!\ external)}}\right]$$

ρ^* = stationary density of active sites

σ^* = stationary branching ratio

Kinouchi & Copelli, *Nat. Phys.* 06

$$P_{ij}(t+1) = P_{ij}(t) + \frac{\epsilon}{NK}(A - P_{ij}) - uP_{ij}(t)\delta(t, t_i)$$

$$\sigma_i = \sum_j^{K_j} P_{ij}$$

$$\frac{\epsilon}{KN}\left(A - \frac{\sigma^*}{K}\right) = \frac{u\sigma^*\rho^*}{K}$$

Mean-field equation (II)

$$\sigma^* = \frac{AK\epsilon}{uKN\rho^* + \epsilon}$$

Mean-field theory

Mean-field equation (I)

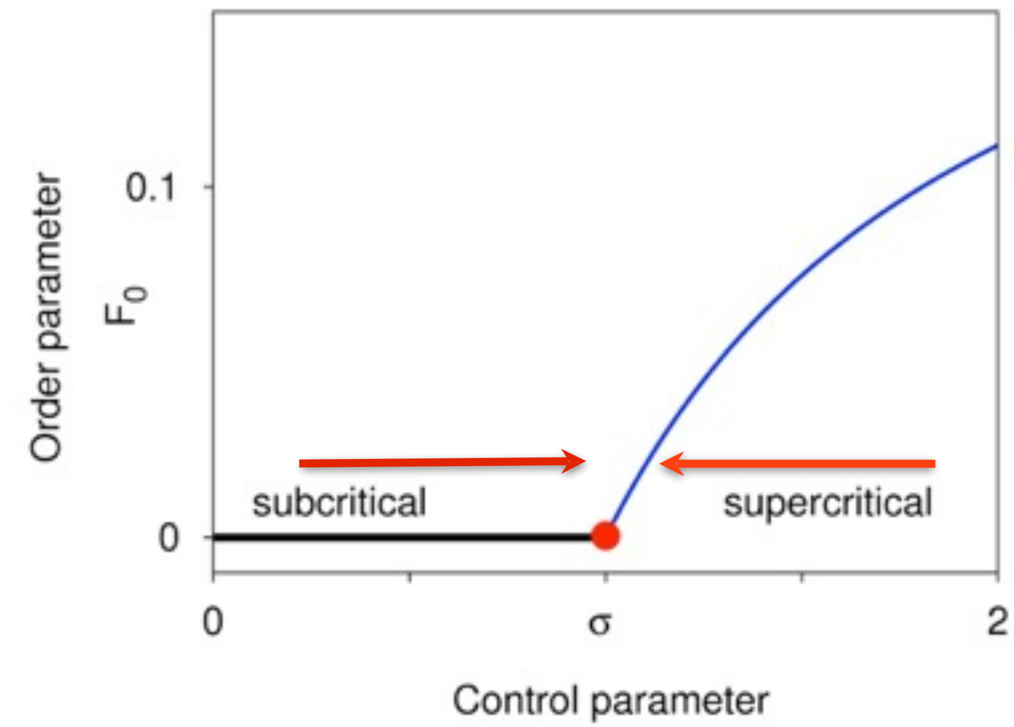
$$\rho^* = \overbrace{(1-(n-1)\rho^*)}^{\mathrm{P(quiet)}} \times \left[1 - \underbrace{(1-\sigma^*\rho^*/K)^K}_{\mathrm{P(!\ from\ neighbors)}} \times \underbrace{(1-p_h)}_{\mathrm{P(!\ external)}}\ \right]$$

Mean-field equation (II)

$$\sigma^* = \frac{AK\epsilon}{uKN\rho^* + \epsilon}$$

ρ^* = stationary density of active sites

σ^* = stationary branching ratio



Near criticality: $\sigma^* \gtrsim 1$

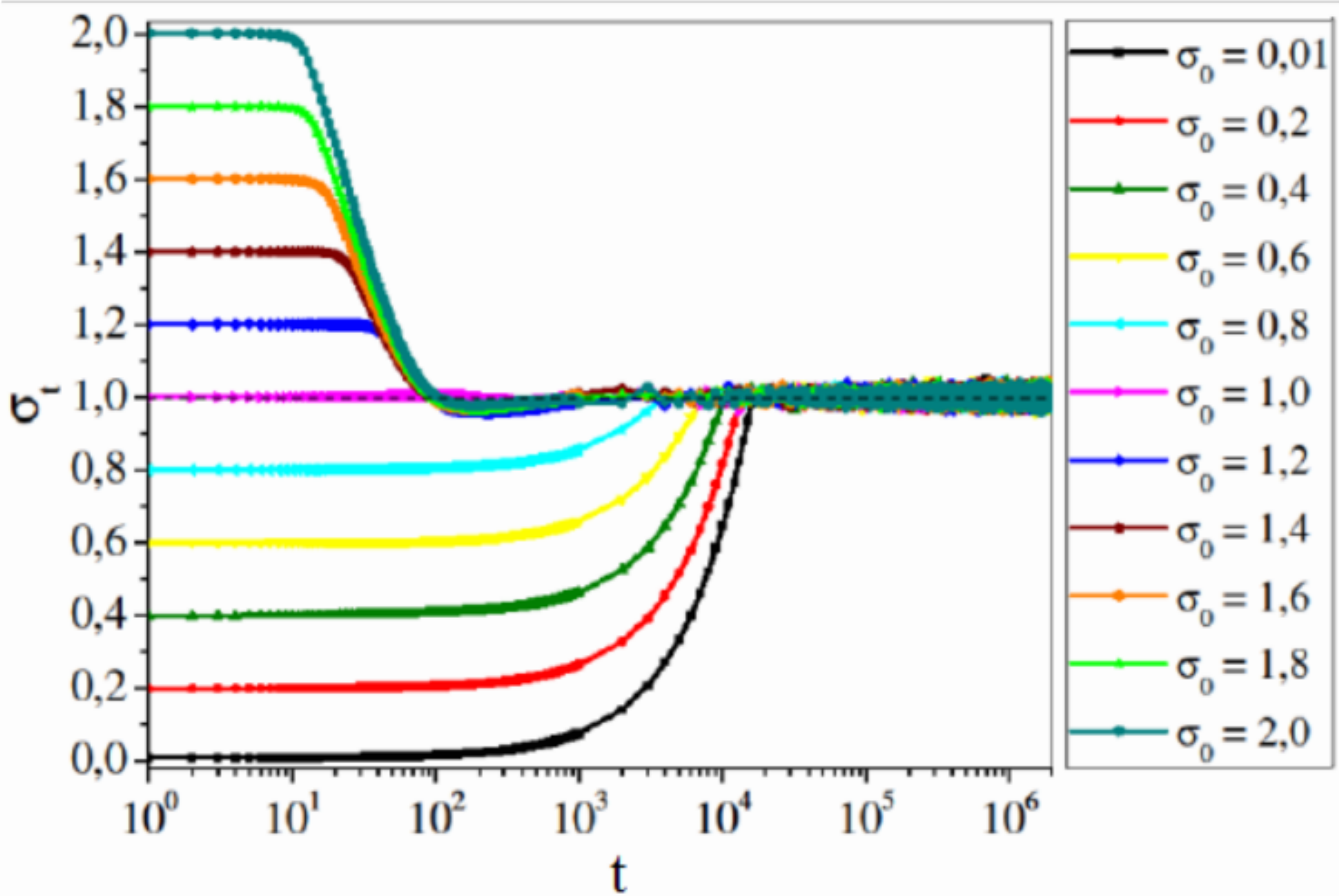
$$\sigma^* - 1 \simeq \frac{(AK-1)}{1 + \left[\frac{uKN}{(n-1)\epsilon}\right]}$$

$\sigma^* \to 1$ when $N \to \infty$

Results

- Analytical (mean-field theory)
- **Simulations**

Branching ratio converges to unity



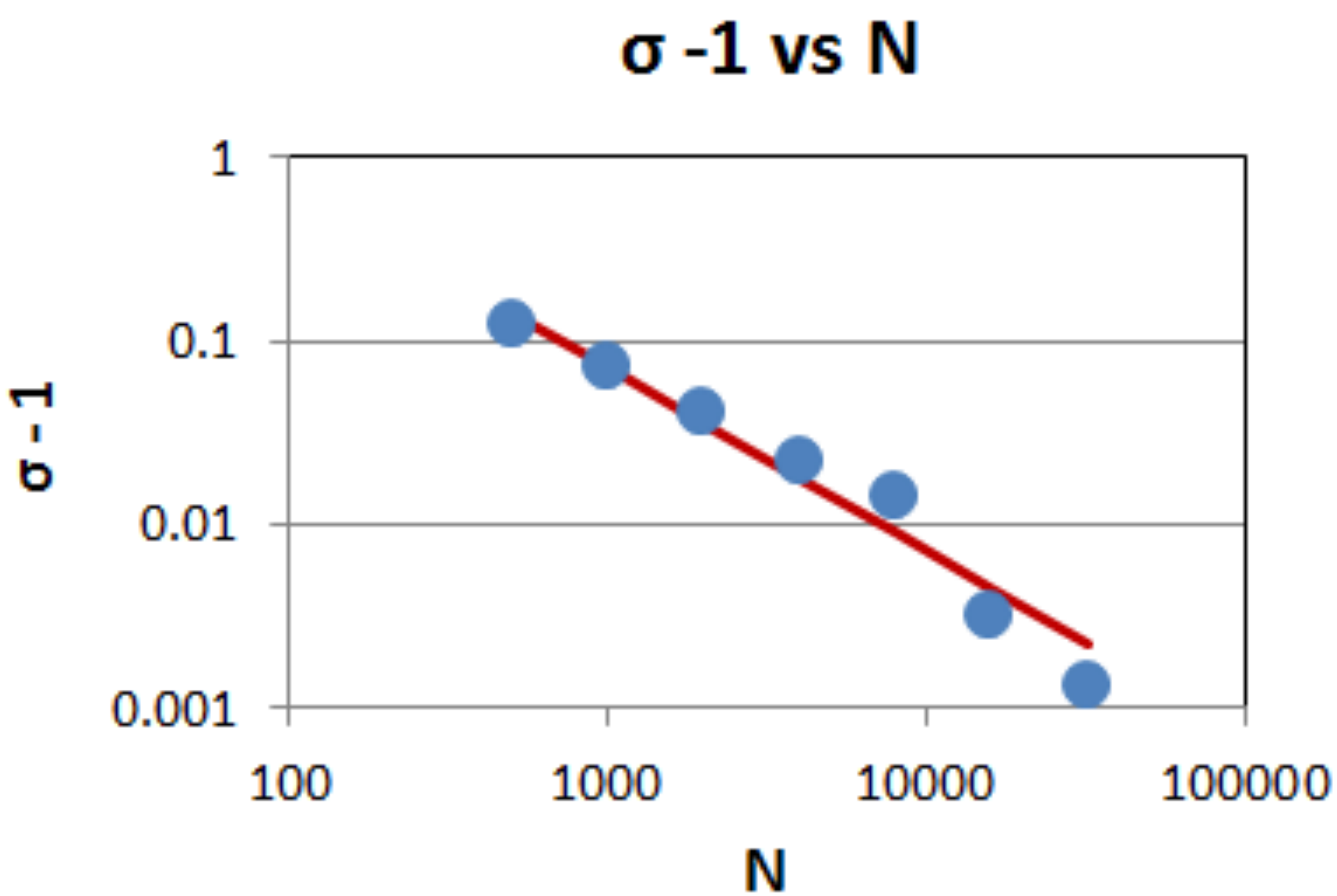
$u = 0.1,\ A = 1.0,\ \epsilon = 2.0$

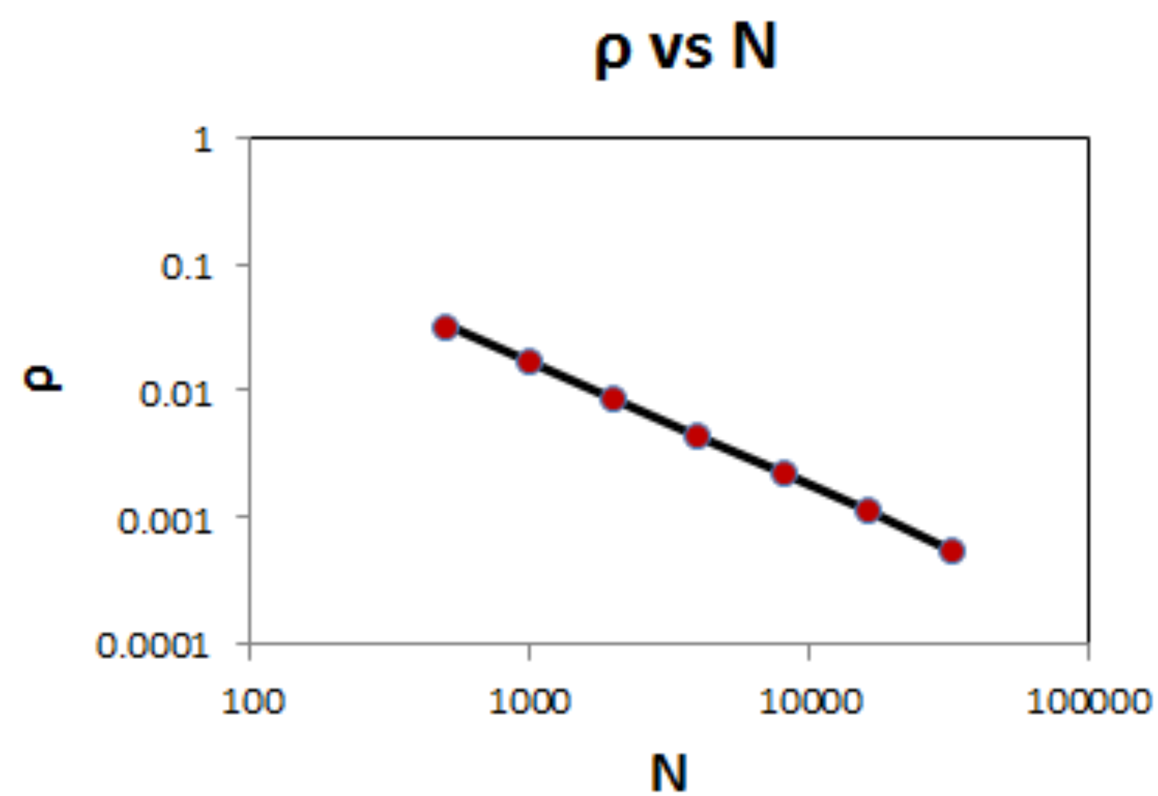
$n = 3,\ K = 10,$ and $N = 30000$

Branching ratio converges to unity

$$\sigma^* - 1 \simeq \frac{(AK-1)}{1+\left[\frac{uKN}{(n-1)\epsilon}\right]}$$

$$\sigma^* = \frac{AK\epsilon}{uKN\rho^* + \epsilon}$$





Branching ratio converges to unity

$$\sigma \to 1$$

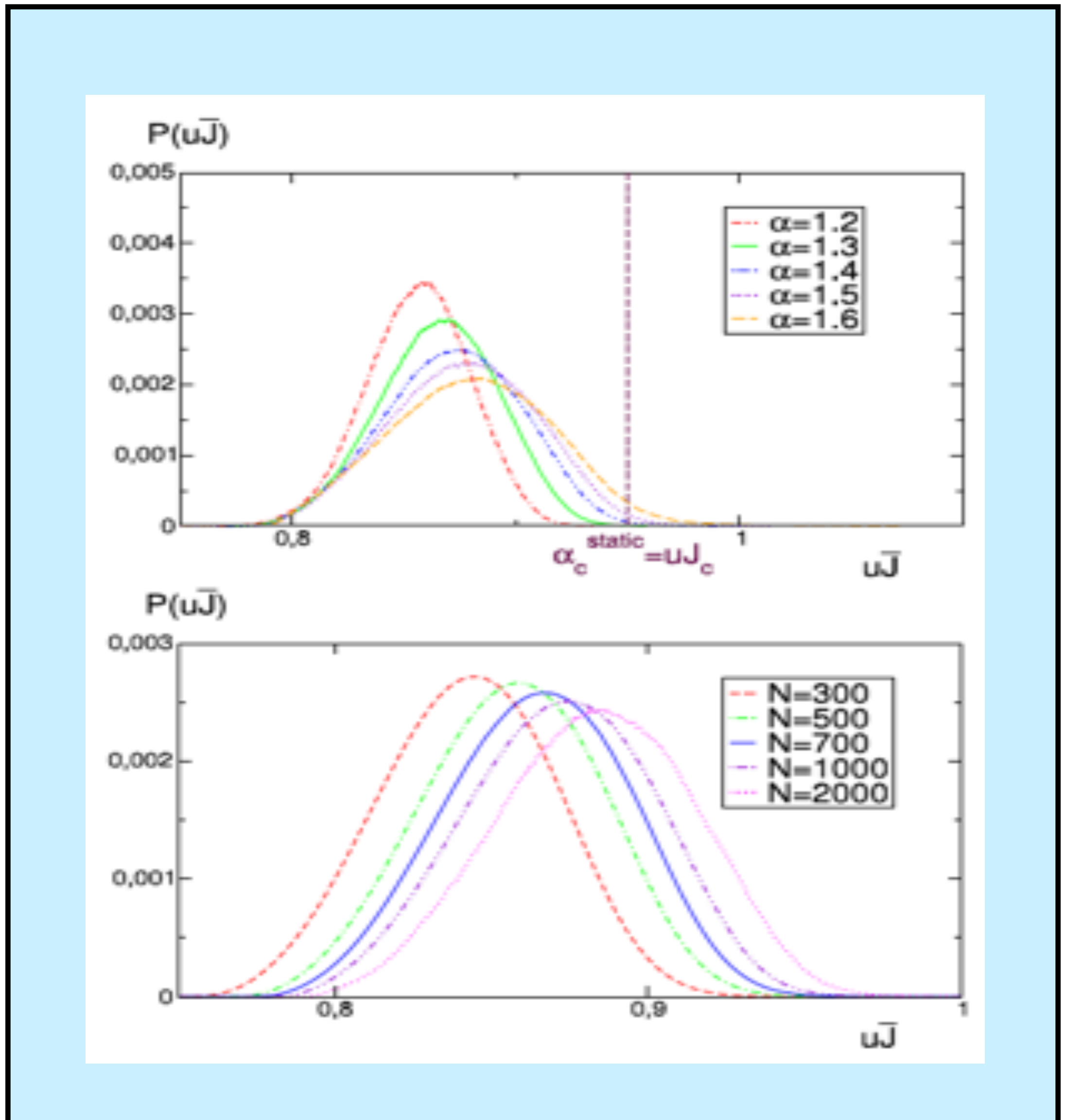
$\Rightarrow$ System becomes conservative, <u>on average</u>!

Is <u>on average</u> good enough?

Fluctuations decrease with system size

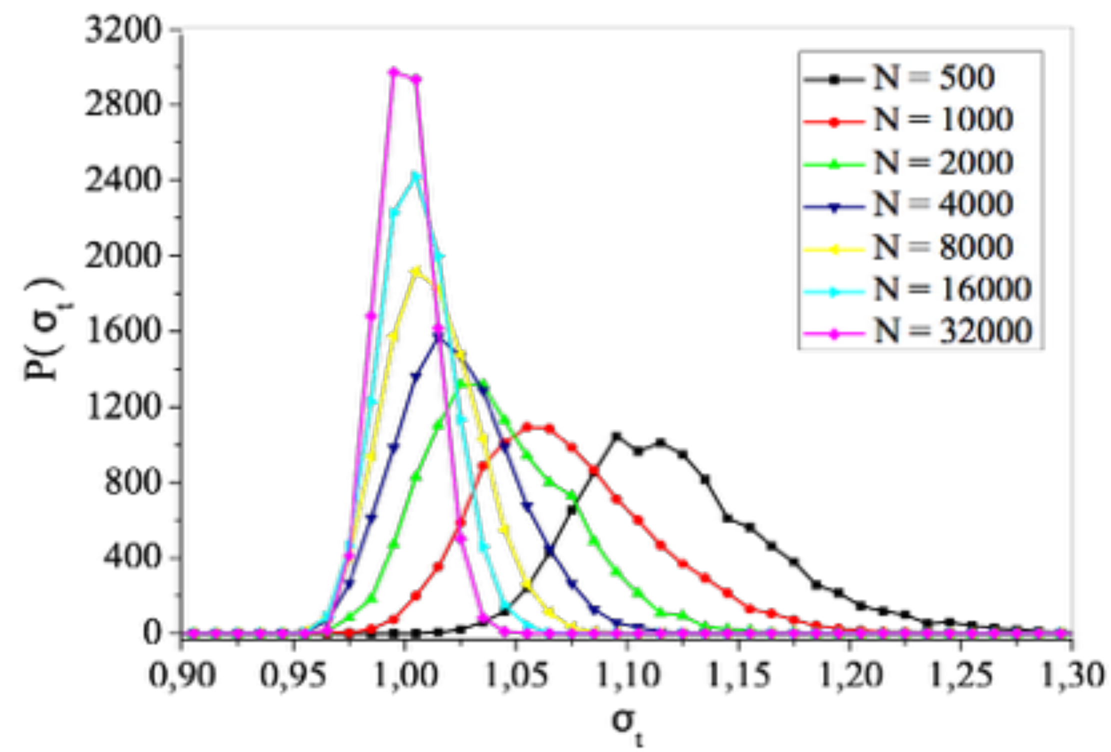
$$P_{ij}(t+1) = P_{ij}(t) + \frac{\epsilon}{NK}(A - P_{ij}) - uP_{ij}(t)\delta(t, t_j)$$

LHG model



Bonachela et al., *JSTAT* 10

Our model

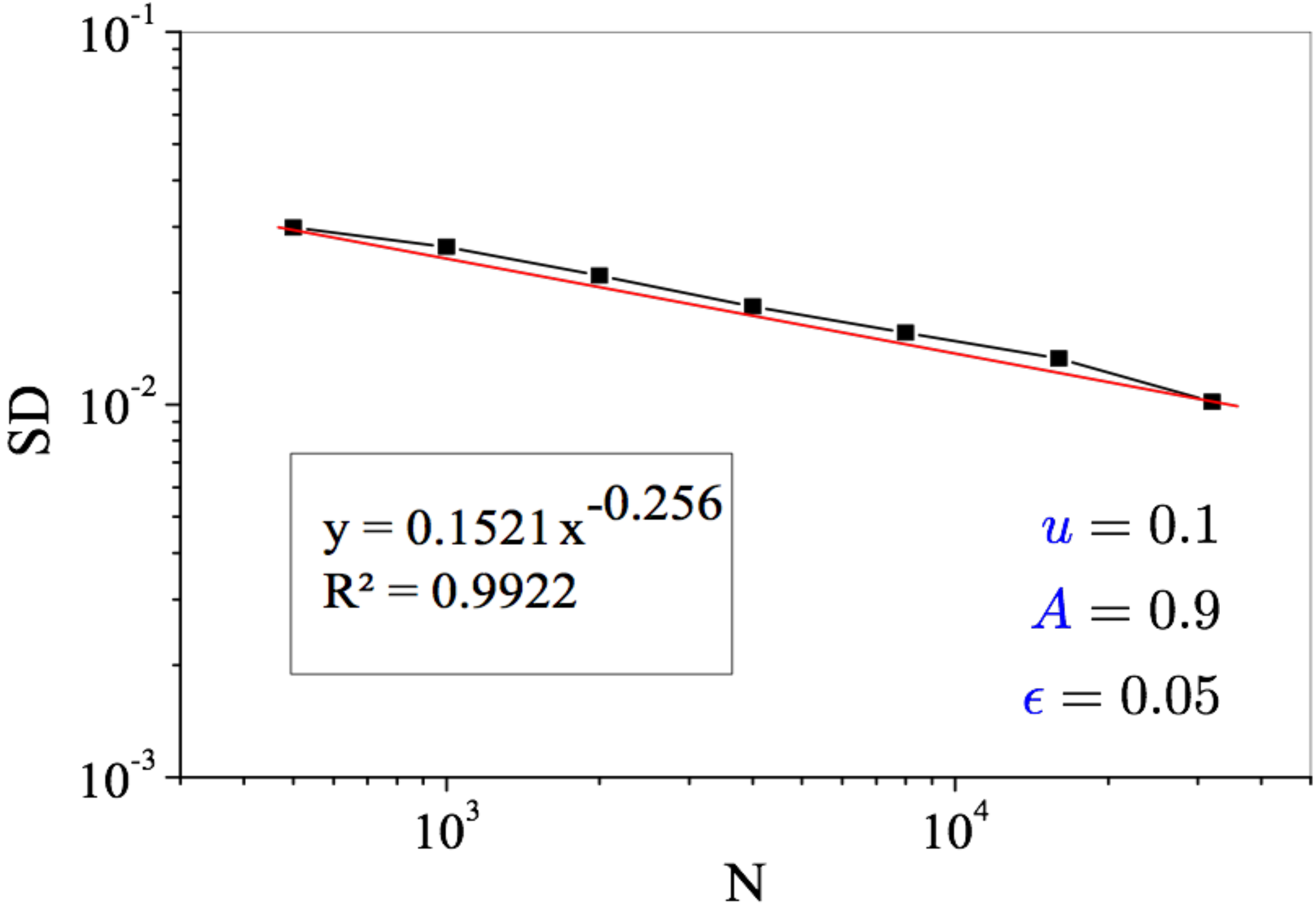


$$u = 0.1$$

$$A = 0.9$$

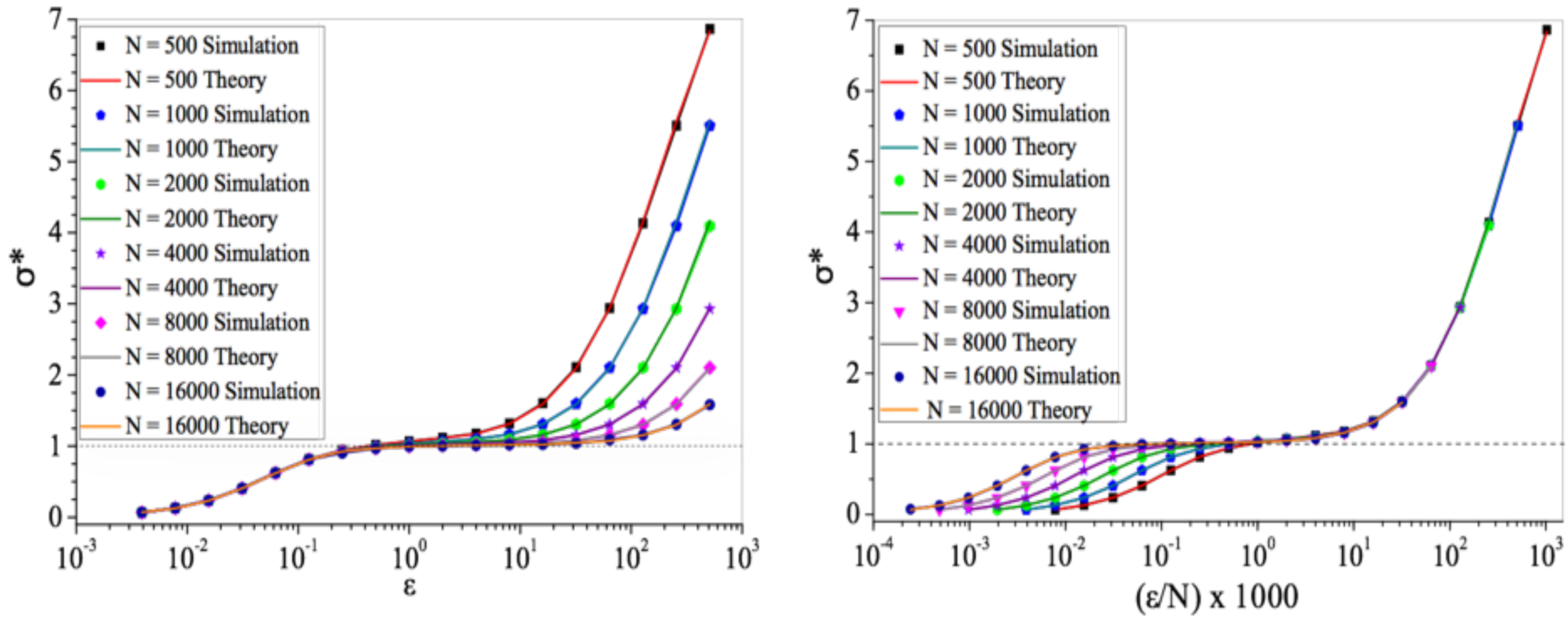
$$\epsilon = 2.0$$

$$\sqrt{\langle \sigma^2 \rangle - \langle \sigma \rangle^2} \sim N^{-1/4}$$



Critical ($\sigma = 1$) region enlarges as N increases

$$\boxed{\sigma^* - 1 \simeq \frac{(AK-1)}{1+\left[\frac{uKN}{(n-1)\epsilon}\right]}} \qquad (\rho \gtrsim 0)$$



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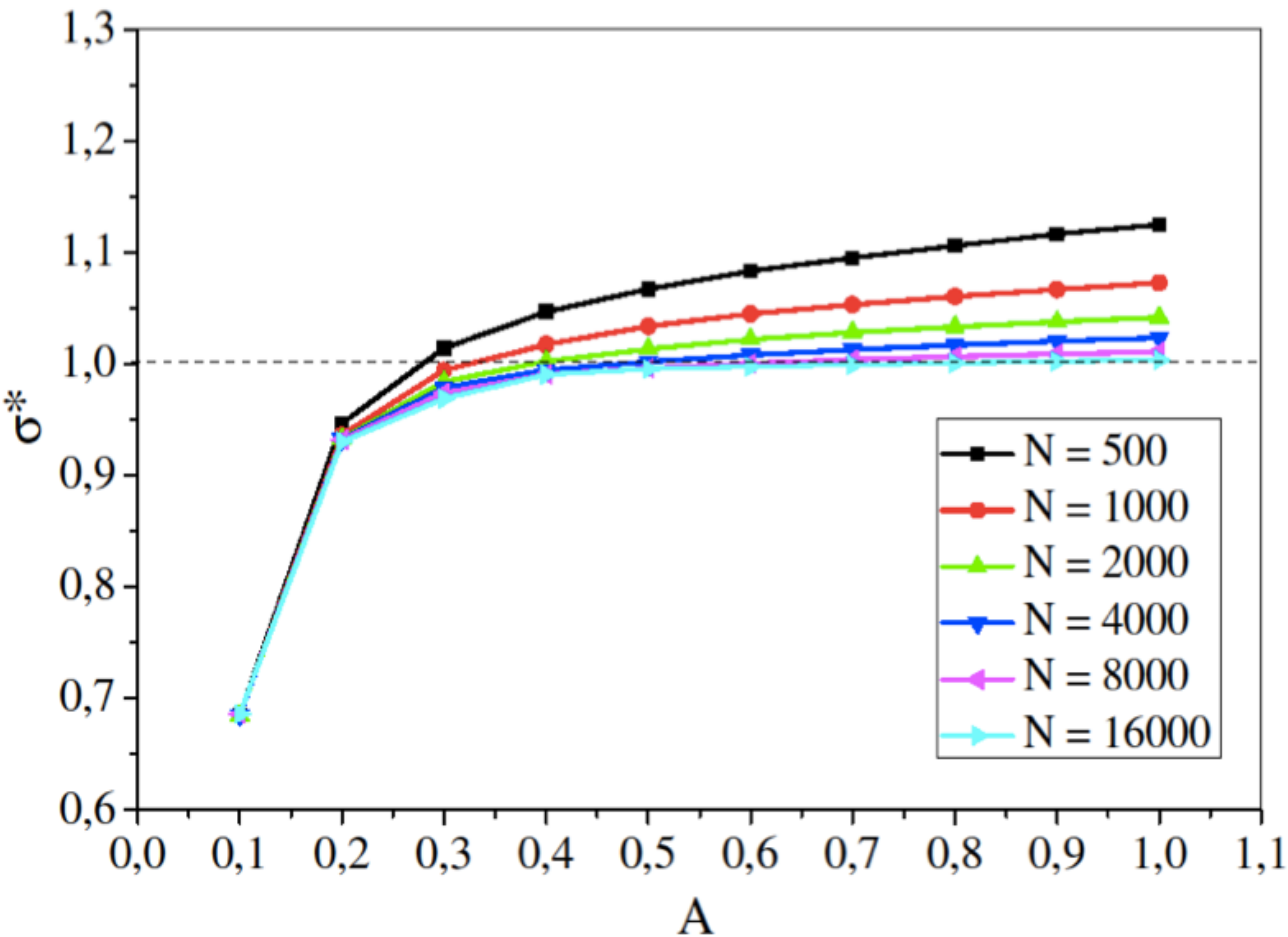

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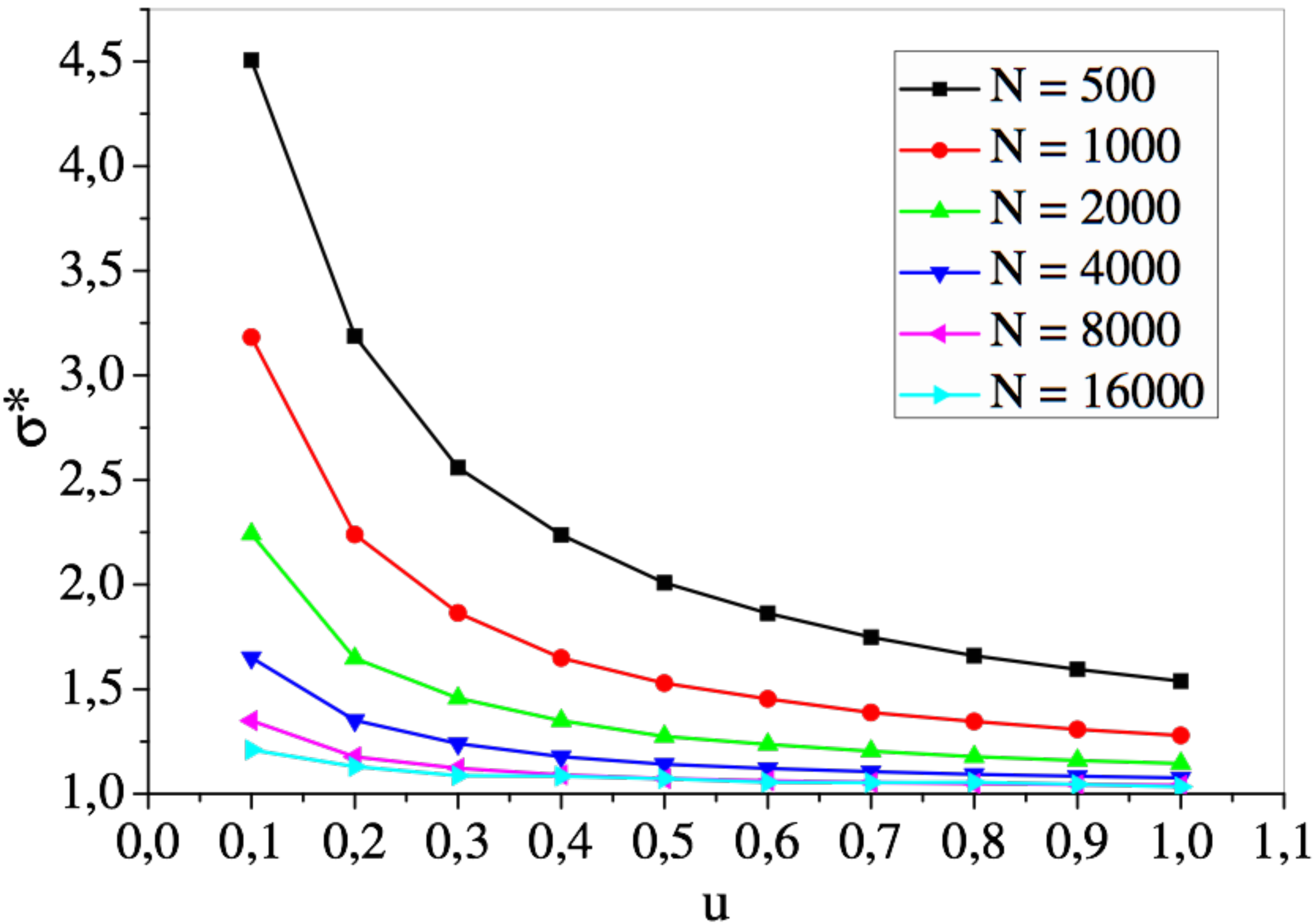
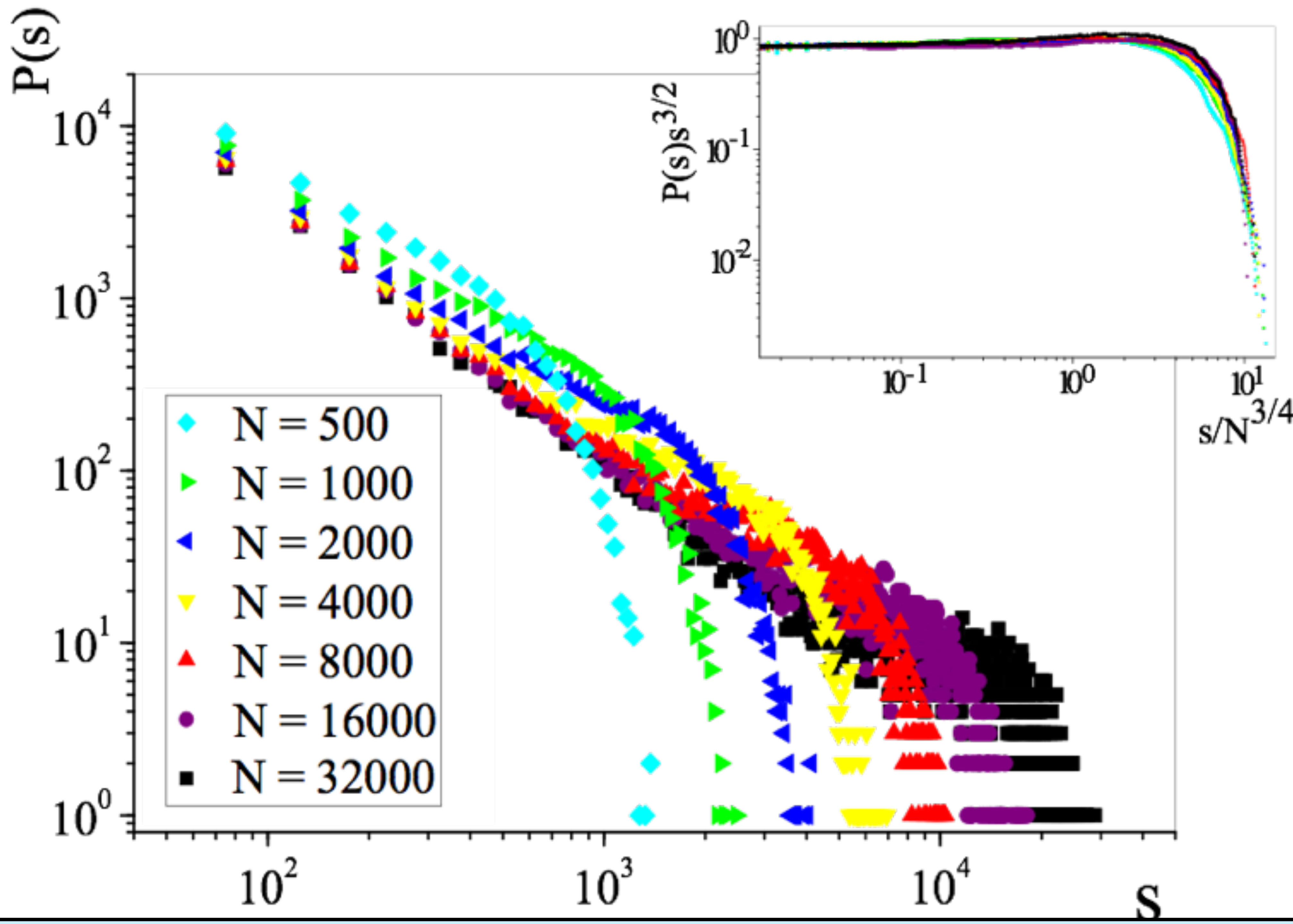


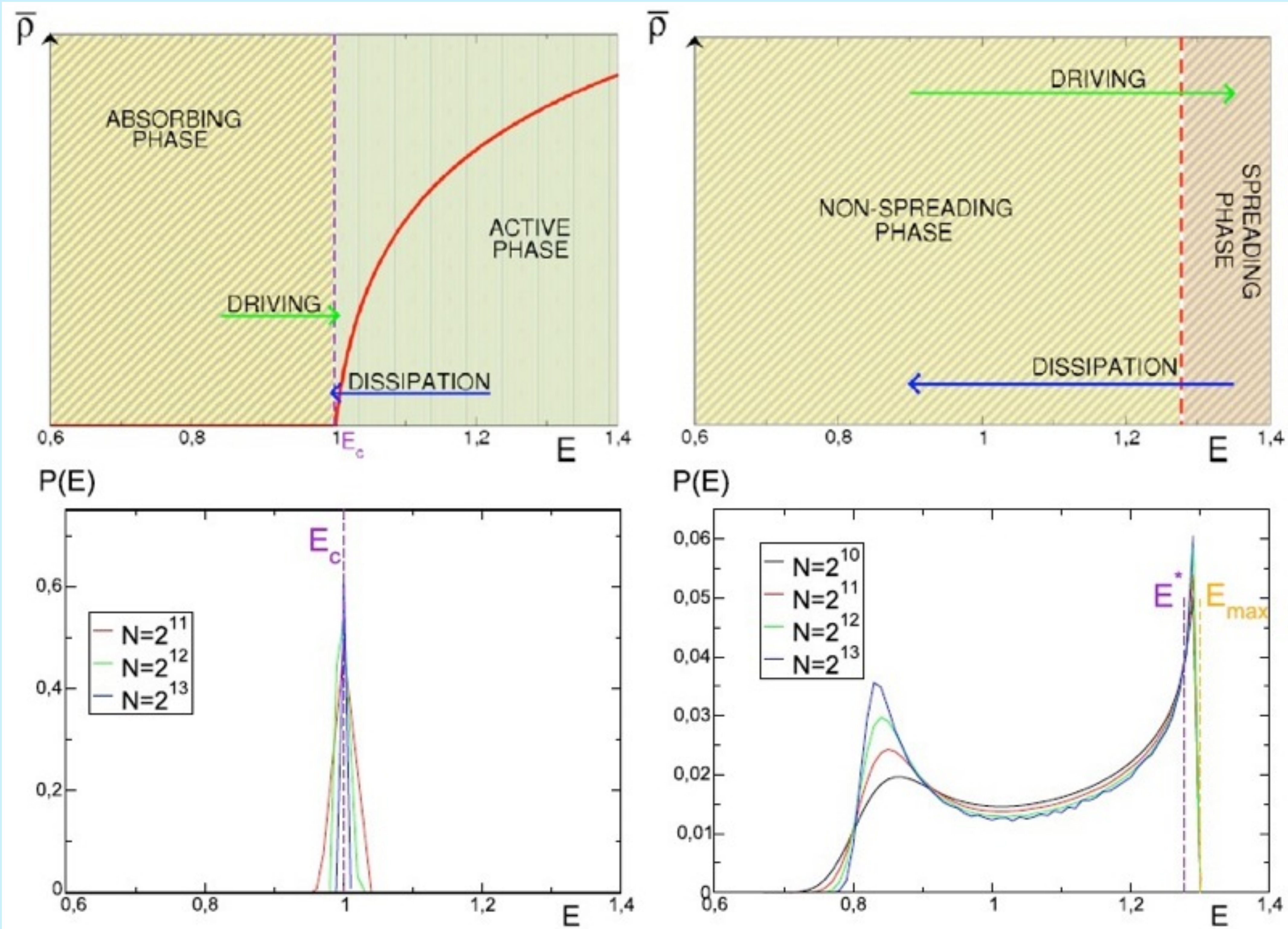
Neuronal avalanches for $\epsilon = \epsilon_0 N^{1/3}$ with cutoff $s_0 \sim N^{3/4}$



Pruessner-Jensen model:

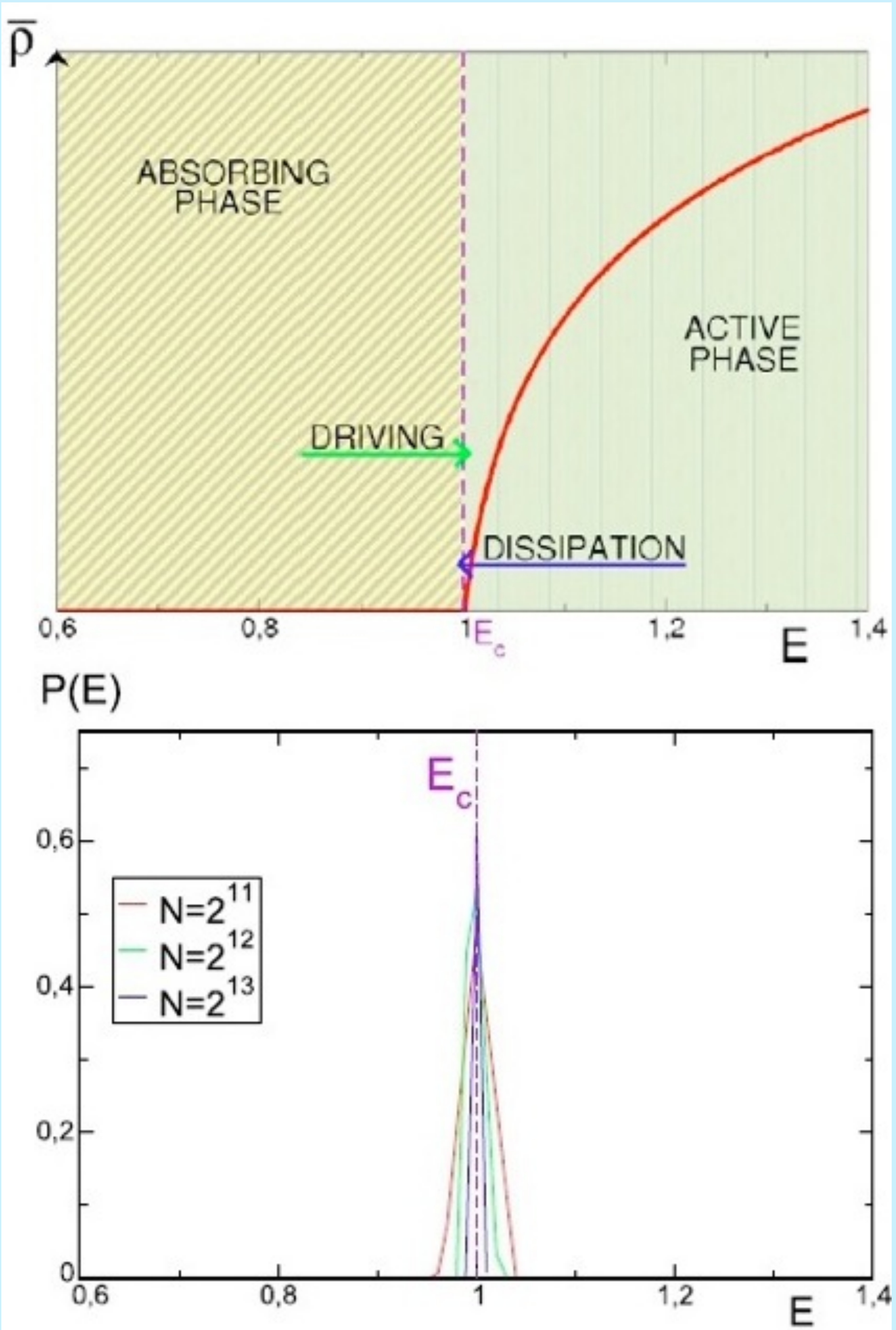
- Similar parameter dependence
- Different cutoff exponent (Bonachela & Muñoz, *JSTAT* 09)

Conservative   Non-Conservative

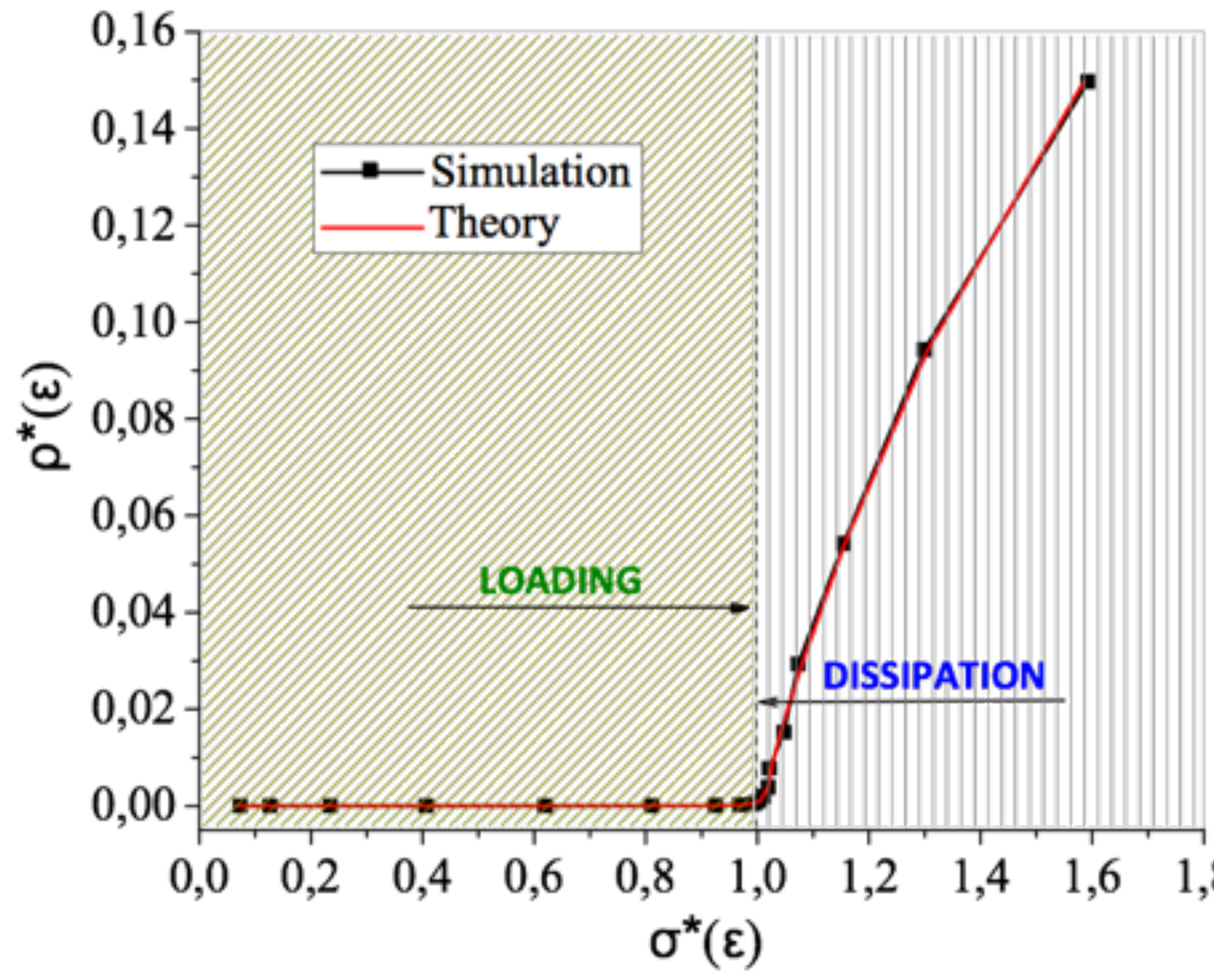


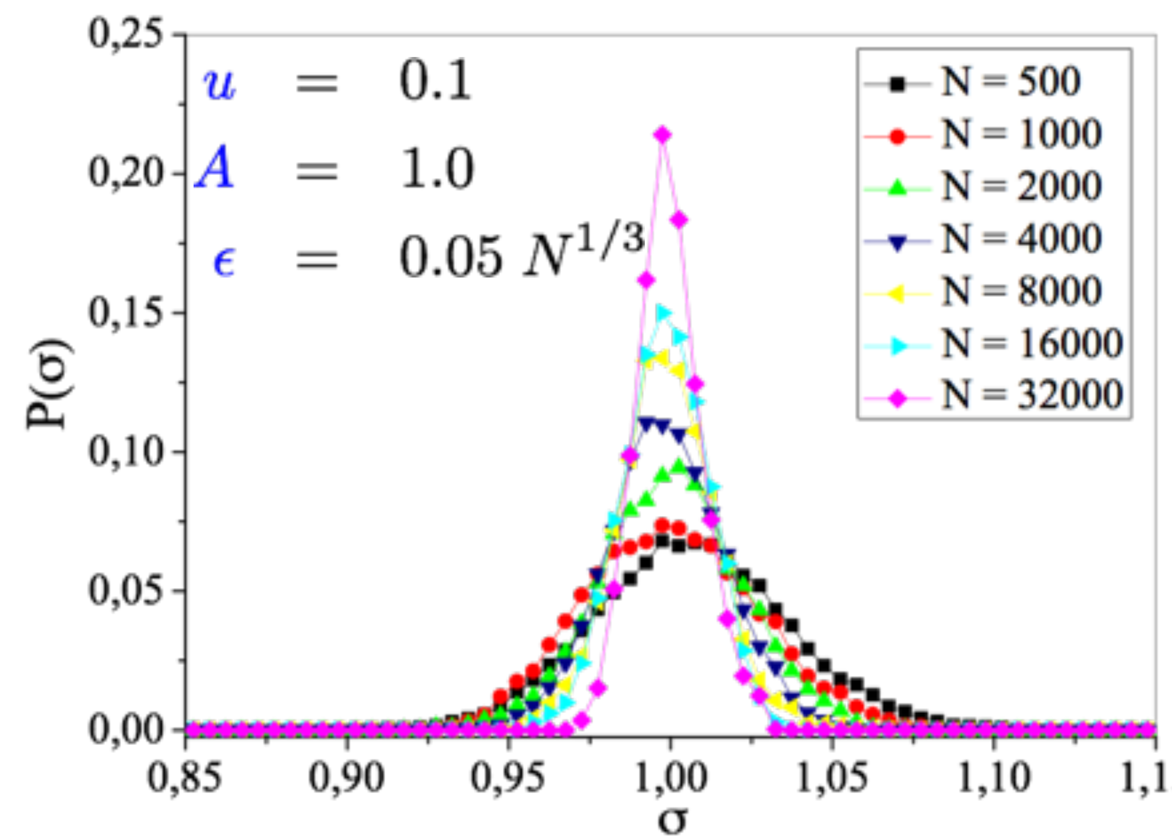
Bonachela & Muñoz, *JSTAT* 09

Conservative



Bonachela & Muñoz, *JSTAT* 09





Conclusions

- $P(\sigma) \overset{N\to\infty}{\to} \delta(\sigma - 1)$, with width decreasing as $N^{-1/4}$
- Employing $\epsilon = \epsilon_0 N^{1/3}$, we have a scaling function for $P(s)$
- Why does it behave so differently from the LHG model?

Possibility:
LHG: $\mathcal{O}(N^2)$ synapses per time step
Here: $\mathcal{O}(NK)$ synapses per time step

If so, should the LHG yield similar results on a random graph?

- Is it bona fide SOC?

Costa, Copelli and Kinouchi – Can dynamical synapses produce true Self-organized criticality?
accepted for publication in *Journal of Statistical Mechanics* (2015)

Perspectives

- Study of neuronal interspike interval (ISI) probability distribution p(t) in the subcritical, critical and supercritical regime

Perspectives

Study of neuronal interspike interval (ISI) probability distribution p(t) in the subcritical, critical and supercritical regime

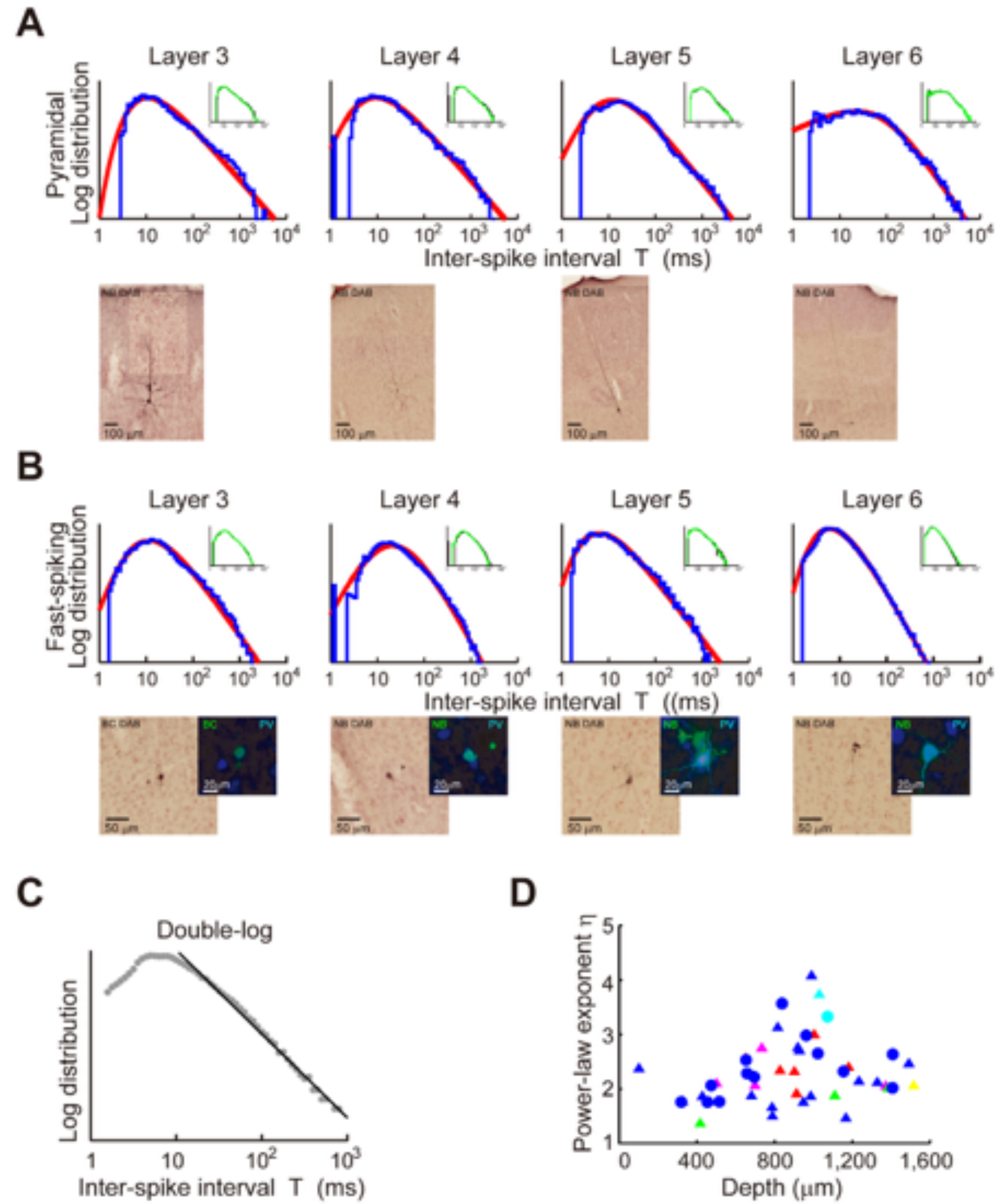
PLoS Computational Biology

Power-Law Inter-Spike Interval Distributions Infer a Conditional Maximization of Entropy in Cortical Neurons

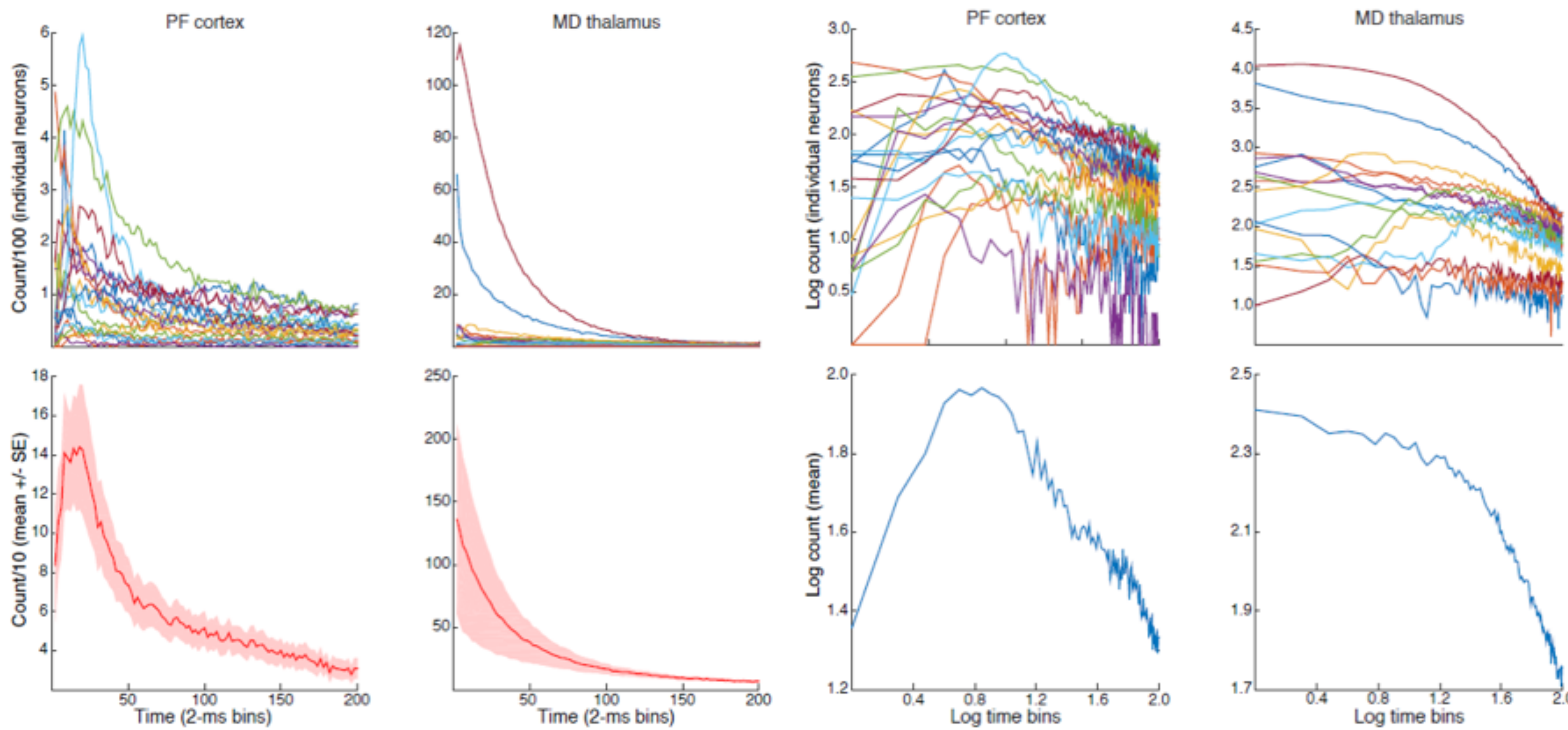
- Yasuhiro Tsubo ,
- Yoshikazu Isomura,
- Tomoki Fukai

- Published: April 12, 2012
- DOI: 10.1371/journal.pcbi.1002461

ISI distributions (log-log graphs)

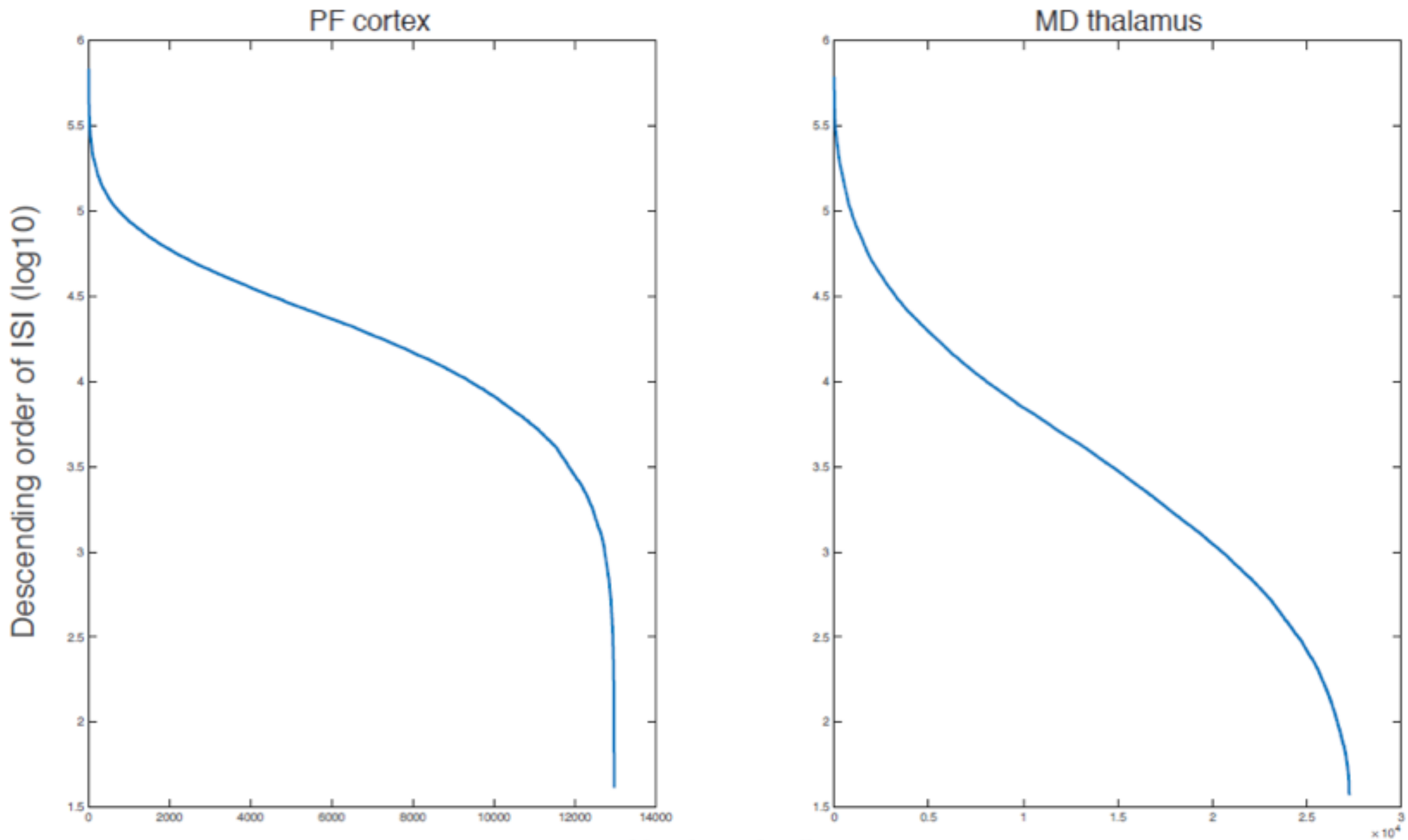


Interspike Intervals (ISI) distributions seems to have long (power laws) tails



Collaboration with Lézio Soares Bueno Júnior started here!

Zipf plot for ISIs

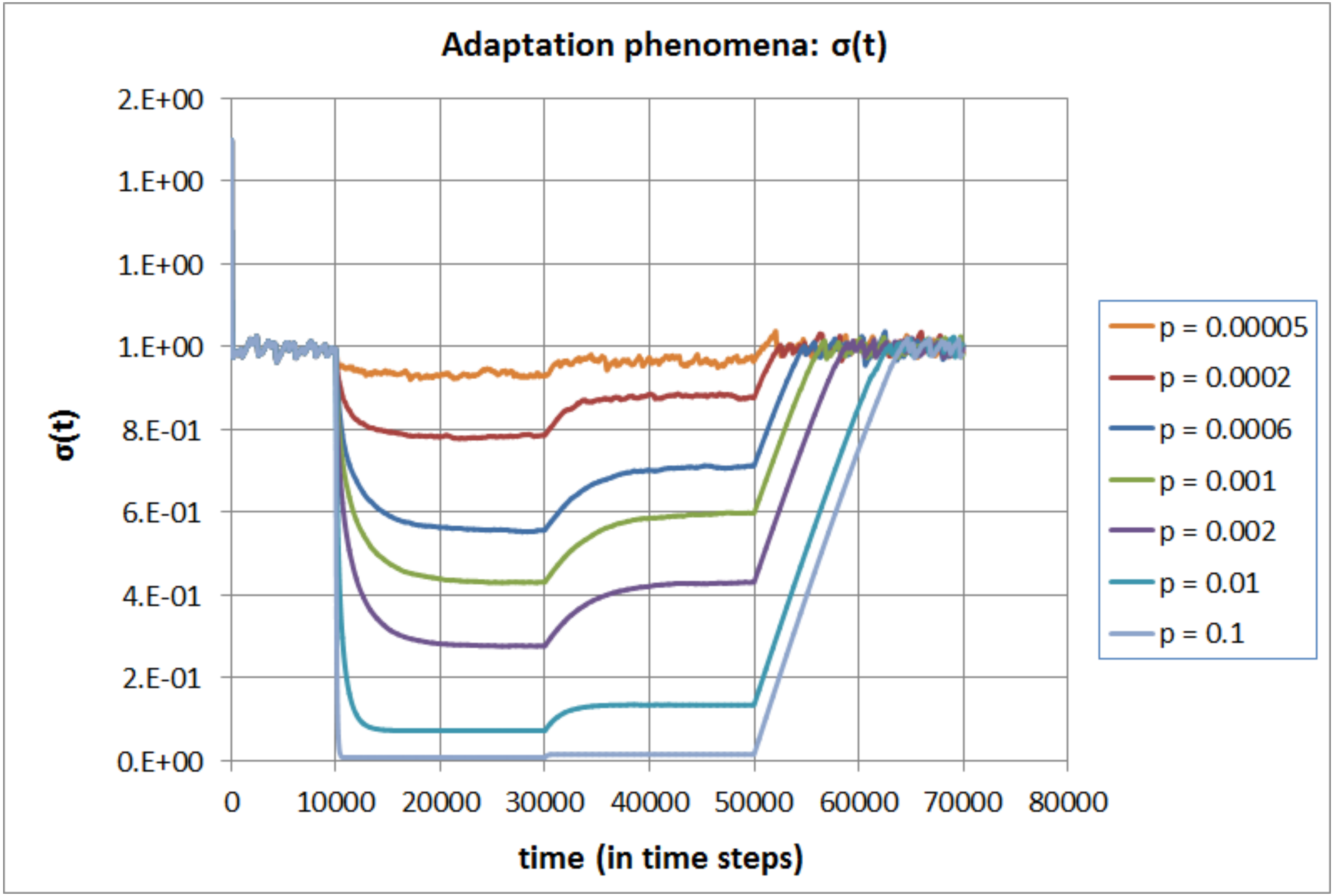


Rank of ISI interval

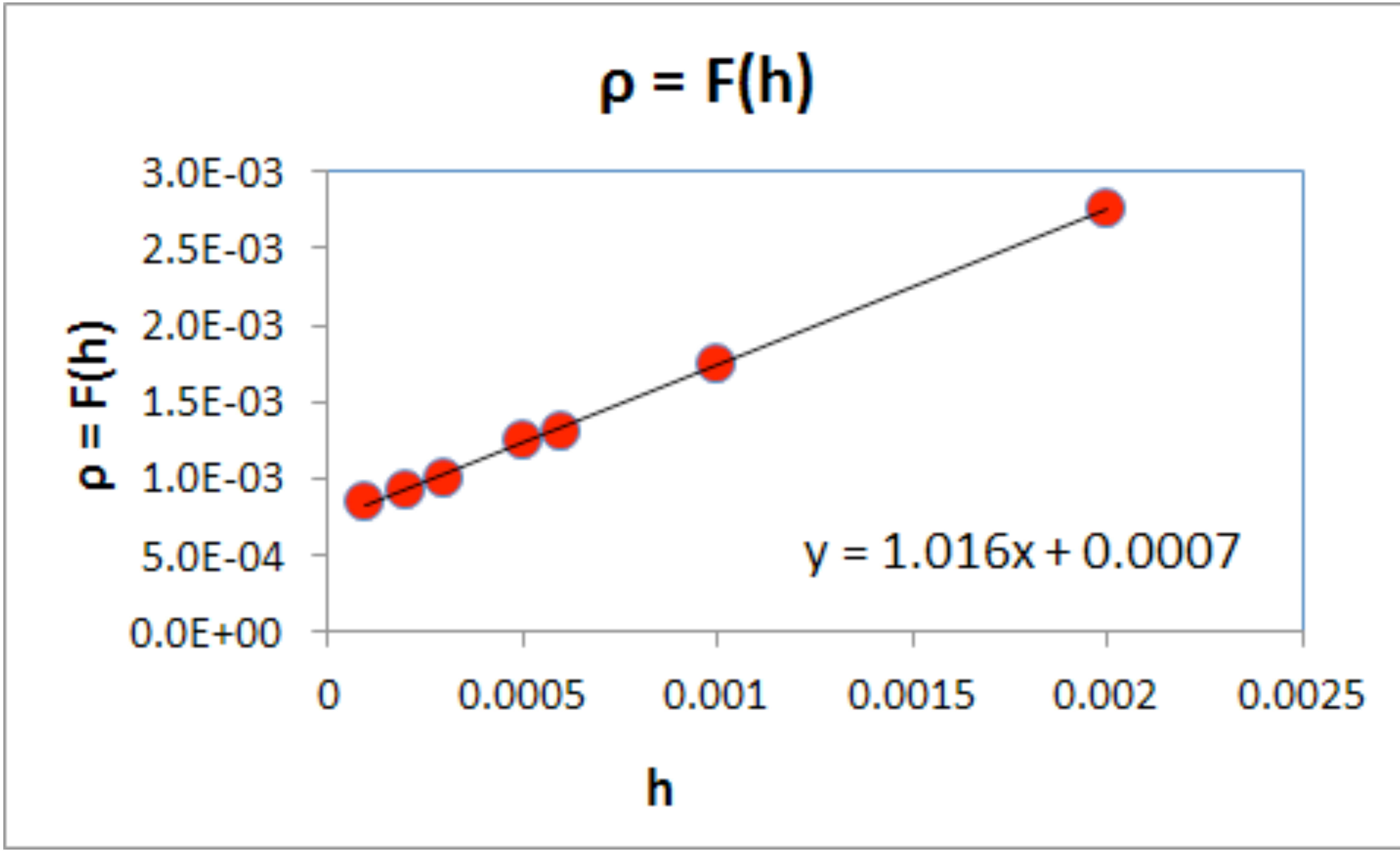
Thank you

Come visit us in Ribeirão Preto!

Adaptation under constant input

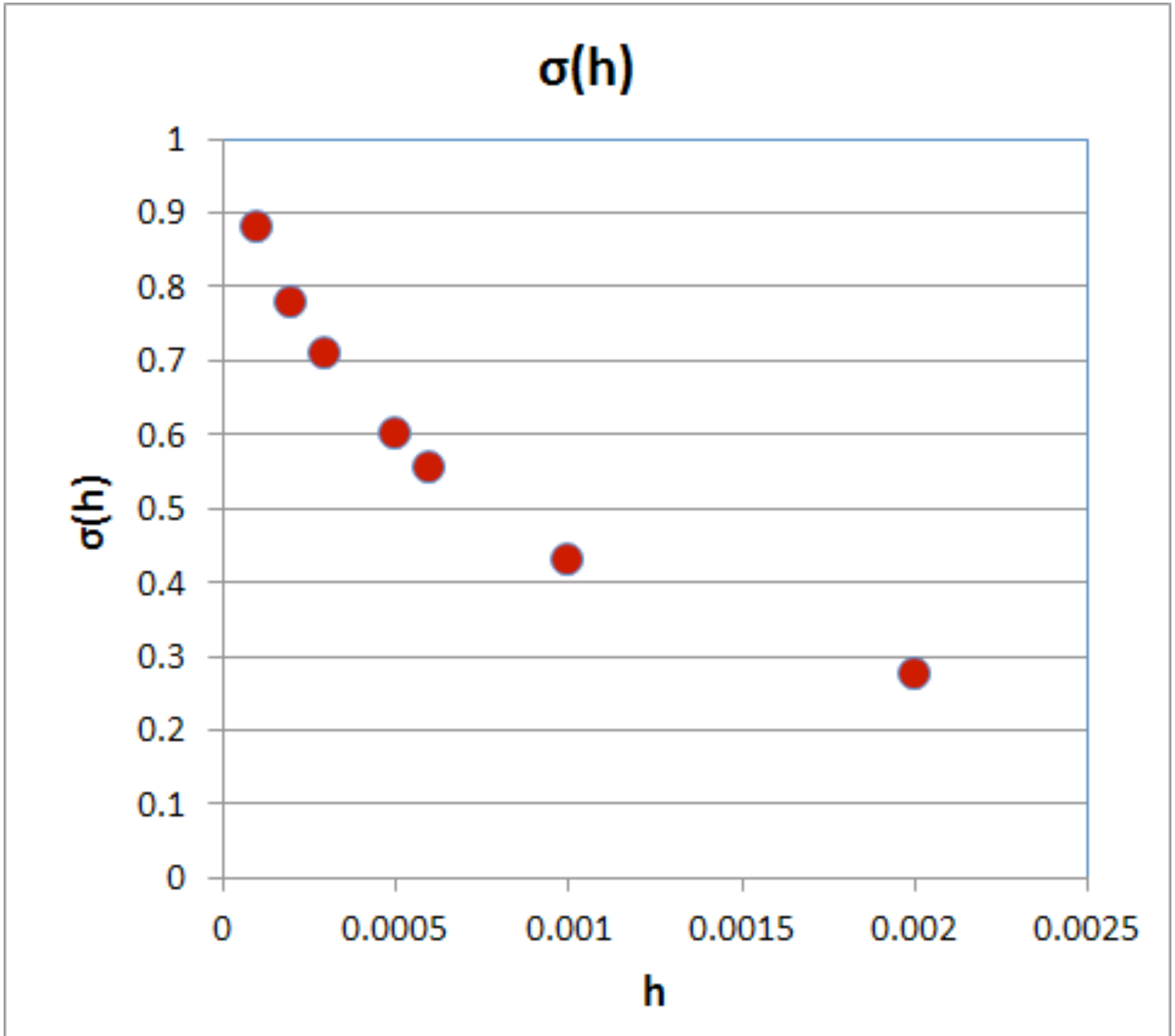


Adaptation



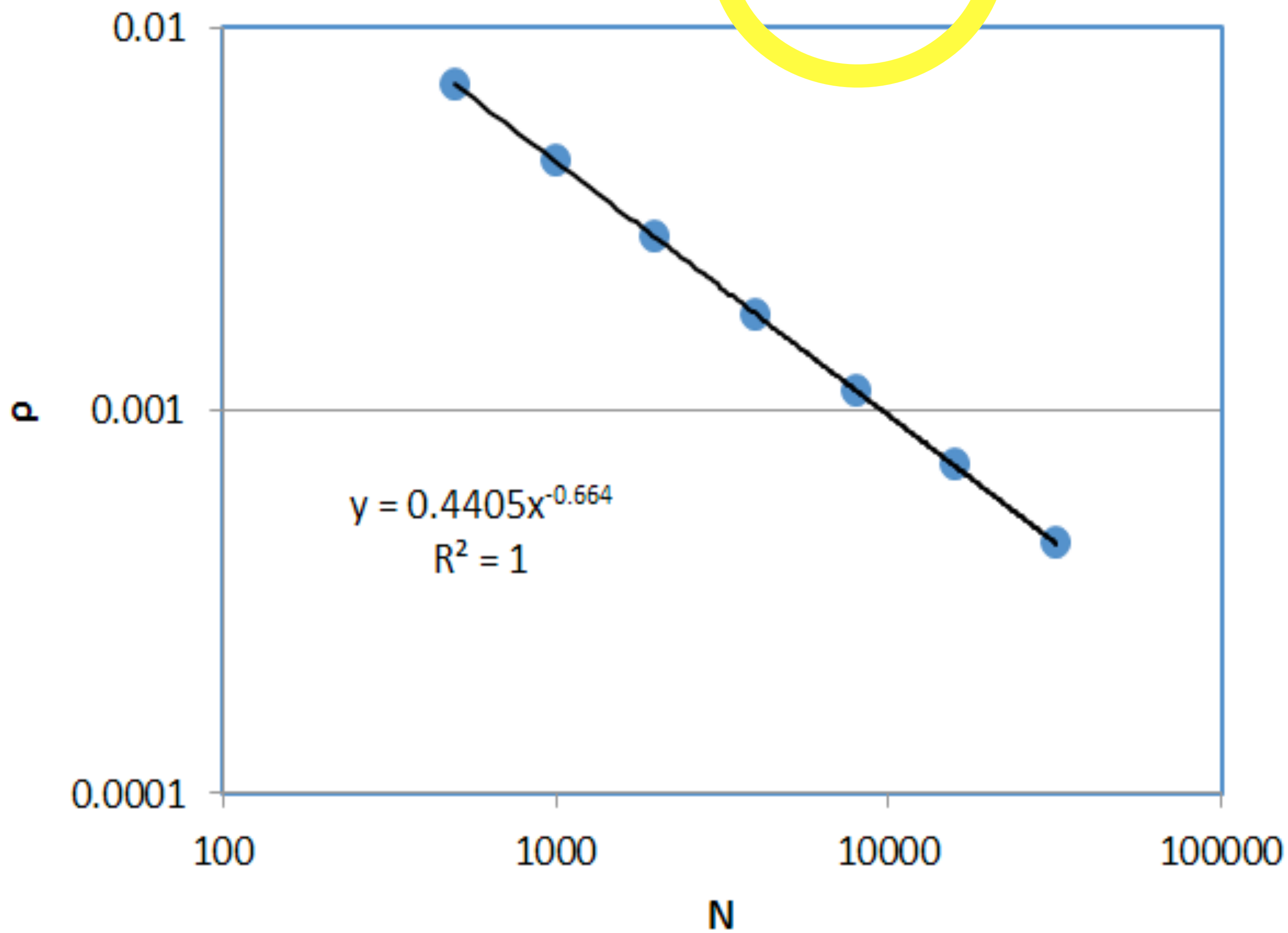
ρ = F(h)
3.0E-03
2.5E-03
2.0E-03
1.5E-03
1.0E-03
5.0E-04
0.0E+00
ρ = F(h)
y = 1.016x + 0.0007
0
0.0005
0.001
0.0015
0.002
0.0025
h

Adaptation



Scaling with system size

Density $\rho \sim N^{-2/3}$



Mean-field equation (II)

$$\sigma^* = \frac{AK\epsilon}{uKN\rho^* + \epsilon}$$

$$\epsilon \simeq \epsilon_0 N^{1/3}$$

A similar scaling was found for the Pruessner-Jensen model (Bonachela & Muñoz, *JSTAT* 09)